2017

MATHEMATICS

PAPER: MAT 404

GROUP A

GENERAL RELATIVITY AND COSMOLOGY II (OPTIONAL IV)

FULL MARKS: 80

TIME: 3 HOURS

The figures in the margin indicate full marks for the questions

1. Attempt any four from the following questions : 10X4=40

(a) What are the conditions required for the static homogeneous universe. Derive the Einstein line element for static homogeneous universe. 3+7=10

(b) Explain the heuristic derivation of Modified Einstein field equation

$$G_{ab} \cong R_{ab} - \frac{1}{2}Rg_{ab} + \Lambda g_{ab} = KT_{ab} ,$$

where symbols have their usual meanings. 10

(c) From the Principle of Cosmology obtain the Robertson-Walker line element. 10

(d) Obtain the dynamical equations of Cosmology

$$\dot{R}^2 + k = \frac{1}{3}K\rho R^2$$

$$\dot{\rho} + 3(p+\rho)\frac{\dot{R}}{R} = 0$$

where symbols have their usual meanings and an over dot denotes d/dt and $\rho = \rho(t)$ and $k=8\pi, G=c=1$. 10

(e) Write a note on the thermal history of the universe. 10

(f) Define Redshift. Using the lumininosity distance d_L of a light source

$d_L = r_1 R(t_0)(1+z_1)$. Derive the redshif versus distance relation as a power series

$$d_L = H_0[z_1 + \frac{1}{2}(1-q_0)z_1^{\,2} + ...]$$

where H_0 and q_0 are Hubble's constant and deceleration parameter at the epoch t_0.

2. Attempt the following questions : 5X6=30

(a) Distinguish between cosmology and cosmological model? Give A comparison of de-Sitter's model universe with actual universe. 5

OR

Show that Einstein's Universe is not an Einstein Space where as de-Sitter's universe is Einstein space.

(b) What is cosmological Constant? Why does Einstein added this term in his field equation? Give the Einstein modified field equation with cosmological term. 5

OR

What is Cosmological Principle? Explain in details.

(c) Explain about motion of test particle in Einstein's Universe. 5

OR

What are points of difference between Einstein's Universe and de-Sitter's universe ?

(d) Explain about Hubble's law and Hubble 's constant. 5

OR

Write a note on Weyl's Postulates. 5

(e) Derive the angular-size versus redshift relation. 5

OR

What are the useful solutions of Friedmann equation.

(f) In a universe with $R(t) \propto t^{2/3}$ and K = 0, a galaxy is observed to have a redshift Z=1.25. How long has light taken to travel from the galaxy to us? 5

OR

Explain about open and closed universe. 5

3. Define the following terms (*Any five*) : 2X5=10

(i) Big-Bang (ii) Big- Bounce (iii) Gravitational Singularity (iv) Isotropic (v) Dark Energy (vi) Radiation dominated era.

GROUP B
FUZZY LOGIC AND CONTROL SYSTEM

1. Answer any four questions : 5X4=20

(a) What is fuzzy logic and Fuzzy Proposition? Write the fundamental difference between classical proposition and fuzzy proposition.

(b) Write about unconditional and unqualified proposition with example.

(c) What is fuzzy quantifier? Explain fuzzy quantifiers of second kind with example.

(d) Let $\tilde{A} = \frac{.1}{x_1} + \frac{.8}{x_2} + \frac{1}{x_3}$, $\tilde{B} = \frac{.5}{y_1} + \frac{1}{y_2}$. Then determine $R(x,y)$.

(e) Write briefly about composition rule of inference.

2. Answer any two questions : 10X2=20

(a) Write an overview of fuzzy expert system.

(b) Define fuzzy implications. Write on S-implication and R-implication. Write the reasonable axioms of fuzzy implications.

(c) Write the steps to discuss the main issues involve in the design of fuzzy controller for stabilizing an inverted pendulum.

3. **Answer any two questions :** 10X2=20

(a) Explain fuzzy controllers with example.

(b) What are defuzzification methods? Explain

(c) Explain on Multiconditional approximate reasoning.

4. **Answer any two questions :** 10X2=20

(a) Write on individual decision making.

(b) Explain on multiperson decision making with example

(c) Solve the following fuzzy linear programming problem

Max $z= 0.5x_1 + 0.2x_2$

Such that

$x_1 + x_2 \leq B_1$

$2x_1 + x_2 \leq B_2, x_1, x_2 \geq 0$

Where

$$B_1(x)=\begin{cases} 1 & ;for\ x \leq 300 \\ \frac{400-x}{100} & ;for\ 300 < x \leq 400 \\ 0 & ;for\ x > 400 \end{cases} \text{ and}$$

$$B_2(x)=\begin{cases} 1 & ;for\ x \leq 400 \\ \frac{500-x}{100} & ;for\ 400 < x \leq 500 \\ 0 & ;for\ x > 500 \end{cases}$$

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