BU/EX/PG/SEM-I/102

2018

MATHEMATICS

MAT 102

ALGEBRA

Full Marks: 80

Time: 3 hours.

1. Answer any *four* from the following questions: 4X5=20

(a) Let G be an infinite cyclic group. Determine $AutG$.

(b) Let G be a non abelian group. Show that the defined map $f:G \to G$ such that $f(x)=x^{-1}$, $\forall x \in G$ is not automorphism. Under what condition this map is automorphism? Justify.

(c) State and prove Sylow's second theorem.

(d) Show that if H be any subgroup of S_n ($n \geq 2$) then either all permutations in H are even or exactly half are even.

(e) Show that number of generators of a finite cyclic group of order n is $\phi(n)$.

2. Answer any *four* from the following questions: 4X5=20

(a) Let L be a left ideal of a ring R and let $\lambda(L)=\{x \in R \mid xa=0, \forall a \in L\}$ then show that $\lambda(L)$ is an ideal of R.

P.T.O.

(b) Let R be a commutative ring with unity. If every ideal of R is prime show that R is a field.

(c) Show that $H_4 = \{4n \mid n \in Z\}$ is a maximal ideal in the ring E of even integers. Is H_4 a prime ideal? Justify.

(d) Prove that an ideal P of a commutative ring R is prime if and only if for two ideals A, B of R, $AB \subseteq P$ implies either $A \subseteq P$ or $B \subseteq P$.

(e) Let $f : R \to R'$ be a homomorphism. Show that Ker $f = (0)$ if and only if f is one-one.

3. Answer any *four* from the following questions: 4X5=20

(a) Let R be a commutative ring with unity and g is a non-zero polynomial in $R[x]$ of degree n with leading coefficient a unit in R. Show that for any $f \in R[x]$ there exist unique polynomials h and r in $R[x]$ such that $f = hg + r$ where either $r = 0$ or deg $r <$ deg g.

(b) Let R be an integral domain with unity. If $l_1 = l.c.m.(a,b)$ in R then l_2 is also an $l.c.m.(a, b)$ if and only if l_1 and l_2 are associates.

(c) Let R be an Euclidean domain and let A be an ideal of R then show that there exist $a_0 \in A$ such that $A = \{a_0 x \mid x \in R\}$.

(d) Let R be a Euclidean domain. For all $a, b \in R$, $a \neq 0, b \neq 0$, there exist t and r in R such that $a = tb + r$ where either $r = 0$ or $d(r) < d(b)$. Show that t and r are uniquely determined if and only if $d(a+b) \leq Max\{d(a), d(b)\}$.

(e) Prove that in a PID an element is prime if and only if it is irreducible. 4X5=20

4. Answer any *four* from the following questions:

(a) Let K be a finite extension of F and L, a finite extension of K. Then show that L is a finite extension of F and $[L : F] = [L : K][K : F]$.

(b) Let $a \in K$ be algebraic over F. Then show that $F(a) = F[a]$.

(c) If L is an algebraic extension of K and K is an algebraic extension of F, then show that L is an algebraic extension of F.

(d) Show that any prime field is either isomorphism to the field of rational numbers or to the field of integers modulo some prime number.

(e) If the rational number r is also an algebraic integer, prove that r must be an ordinary integer.

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