2018

MATHEMATICS

MAT 103

DIFFERENTIAL EQUATIONS

Full Marks : 80

Time : 3 hours

The figure in the margin indicate full marks for the questions

1. Answer any four from the following questions 5X4=20

(a) Apply Picard's method to solve the initial value problem upto third approximation $\frac{dy}{dx} = e^x + y^2, (y) = 0$

(b) Prove that the solution space $S_C = \{y \in C^2[a,b]: y^{//} + P y^{/} + Qy = 0, x \in [a,b]\}$ is vector space with dimension 2.

(c) Apply methods of variation of parameter to solve : $y_2 - 2y_1 = e^x \cos x$

(d) Show that linearly independent solution of $y_2 - 2y_1 + 2y = 0$ are $e^x \sin\, x$ and $e^x \cos x$. What is general solution ? Find the solution $y(x)$ with the property $y(0) = 2, y_1(0) = 3$

(e) Let $f(x) = x^3$ and $g(x) = x^2|\,x\,|$ for $x\epsilon[-1,1]$ then

i. Show that f and g are linearly independent.

ii. $W(f,g) = 0\,;\ x\epsilon[-1,1]$

iii. From i and ii what you can conclude.

2. Answer any two from the following questions : 10X2=20

i. Solve : $(1 - x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$ in series.

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ii. Solve : $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-4)y=0$ in series.

iii. Find all the eigen values and eigen functions of the Strum-Liouville problem $\frac{d^2y}{dx^2}+\lambda\frac{dy}{dx}=0$ with with $(y)_{x=0}+(\frac{dy}{dx})_{x=0}=0$ and $(y)_{x=1}+(\frac{dy}{dx})_{x=1}=0$.

3. Answer any four from the following questions 5X4=20

i. Solve : $z(z^2+xy)(px-qy)=x^4$

ii. Solve :

$\{(b-a)/a\}yzp+\{(c-a)/a\}zxq=\{(a-b)/c\}xy$

iii. Find a complete and singular integrals of

$2xz-px^2-2qxy+pq=0$

iv. Find the integral surface of the partial differential equation $(x-y)p+(y-x-z)q=z$ through the circle

$z=1,\ x^2+y^2=1$.

v. Reduce $\frac{\partial^2z}{\partial x^2}-\frac{\partial^2z}{\partial x^2}=0$ to canonical form.

4. Answer any two from the following questions 10X2=20

i. Obtain the solution of Laplaces's equation in spherical polar coordinates.

ii. Obtain solution of heat equation in cylindrical polar coordinates.

iii. Obtain the solution of wave equation in cylindrical coordinates by the method of separation variables.