2018

MATHEMATICS

MAT 104

CLASSICAL MECHANICS

Full Marks: 80

Time: 3 hours

The figures in the margin indicate full marks for the questions

Attempt the following questions:

1. (a) Derive Euler-Lagrange equation. 10

Or,

Find the path $y = f(x)$ connecting A and B which Minimizes the time travelled from A to B under the influence of gravity.

(b) State Hamilton's principle. Deduce Lagrange's equation from Hamilton's principle. 10

Or,

Derive the equation for time period for a simple pendulum using Lagrange's equation.

2. Write short notes on : 2X5=10

(a) Degrees of freedom

(b) Constraints

(c) Holonomic system

(d) Generalized velocity

(e) Scleronomic system

3. Derive the Lagrange's equation of motion of second kind 10

4.(a) What are canonical transformations? Write a note on the Bilinear invariant as condition for a transformation to be canonical. 6

(b) Using the invariant of bilinear form, show that the transformation

$Q = \frac{1}{p}$ and $P = qp^2$ is canonical. 4

Or

Test whether the following transformation is canonical

$$Q = \log 1 + \sqrt{q}\,Cosp,$$

$$P = 2\sqrt{q}\,Sinp(1 + \sqrt{q}\,Cosp),$$ 4

5 (a) Deduce the relation between the Lagrange and Poisson brackets. 6

(b) Show that the transformation defined by

$$q = \sqrt{(2P}\sin Q,$$

$$p = \sqrt{(2P}\cos Q,$$

is canonical by using Poisson's bracket conditions. 4

OR

If $[\varphi,\psi]$ be the Poisson bracket, then prove that

$$\frac{\partial}{\partial t}[\varphi,\psi] = [\frac{\partial\varphi}{\partial t},\psi] + [\varphi,\frac{\partial\psi}{\partial t}]$$ 4

6 (a) Apply the Hamilton-Jacobi method to determine the motion of a body falling vertically in a uniform gravitational field. 6

(b) Show that Lagrange bracket does not obey the commutative law of algebra. 4

OR

Show that the Lagrange bracket is invariant under canonical transformation. 4

7. (a) Derive the Hamilton-Jacobi equation. 6

(b) Explain about the separation of variables in Hamilton-Jacobi equation. 4

OR

Write the conditions for a transformation to be canonical. 4

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