2018

MATHEMATICS

MAT 105

TENSOR ANALYSIS AND LINEAR ALGEBRA

Full marks: 80

Time: 3 hour

The figures in the margin indicate full marks for the questions

Answer Group A and Group B in separate answer books

Group A: Tensor analysis

1. Answer any *four* of the following questions: $4 \times 5 = 20$
   (a) Define Kronecker delta $\delta^i{}_j$ and prove that it is a mixed tensor. State its rank and order.
   (b) Show that every tensor can be expressed in terms of symmetric and a skew-symmetric tensor.
   (c) Distinguish between a symmetric and an anti symmetric tensor.
   (d) Write the transformation formula for covariant, contravariant and mixed tensor of rank two.
   (e) What do you understand by fundamental tensors, associated tensor, and tensor densities?
2. Answer any *two* of the following questions $2 \times 5 = 10$
   (a) Evaluate the Christoffel's symbols of both kinds for spaces, where $g_{ij} = 0$ if $i \neq j$.
   (b) Calculate the non vanishing Christoffel's symbols corresponding to the metric
   $ds^2 = (dx^1)^2 + (x^1)^2(dx^2)^2 + (x^1)^2 sin^2 x^2 (dx^3)^2$.
   (c) If A^i is a contravariant vector, then to prove that $divA^i = \frac{1}{\sqrt{g}}\frac{\partial}{\partial x^i}(A^i\sqrt{g})$ where $g = |g_{ij}|$.

 P.T.O.

3. Find the covariant derivative of covariant tensor of second order. 10

Group B: Linear Algebra

4. Answer any *four* from the following questions: $4 \times 5 = 20$

(a) Let T be a linear operator on V and let $Rank T^2 = Rank T$ then show that $Range T \cap Ker T = \{0\}$.

(b) Find the range, rank, Ker and nullity of the following linear transformation, $T: R^2 \to R^3$ such that $T(x_1, x_2) = (x_1, x_1 + x_2, x_2)$.

(c) Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 2 & -1 & 0 \\ 3 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

(d) Determine whether $A = \begin{bmatrix} 2 & -3 \\ 1 & -2 \end{bmatrix}$ is diagonalizable.

(e) Let $B = \{u_1, u_2, u_3, \ldots, u_m\}$ be basis for a subspace U of an n-dimensional vector space V. Show that U is an invariant subspace under the linear transformation $T: V \to V$ if and only if $T(u_i) \in U$ for $i = 1, 2, 3, \ldots, m$. Determine whether the subspace $M = span\left\{\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \\ 0 \\ 0 \end{bmatrix}\right\}$ is invariant under

$$T\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} a+b-d \\ b \\ c+d \\ d \end{bmatrix}.$$

5. Answer any *four* from the following questions: $4 \times 5 = 20$

(a) Let V be an inner product space. Then show $|(u,v)| \leq \|u\| \|v\|$ for all $u, v \in V$.

(b) Obtain an orthogonal basis w.r.t. the standard inner product for the subspace of R^3 generated by (1, 0, 3) and (2, 1, 1).

(c) Let S be an orthogonal set of non-zero vectors in an inner product space V. Then show that S is a linearly independent set.

(d) A linear transformation $T: V \to V$ is a projection if and only if $T = T^2$.

(e) Let T be a linear operator on a finite dimensional complex inner product space V. Then prove that $T = T_1 + iT_2$ for some uniquely determined self-adjoint linear operators T_1, T_2 on V.

****

P.T.O.