2018

MATHEMATICS

MAT 205

CONTINUUM MECHANICS AND LATTICE THEORY

Full Marks: 80

Time: 3 hours

The figures in the margin indicate full marks for the questions

(Group A- Continuum Mechanics)

1. Attempt any *four* from the following questions 5X4=20

(a) Explain Cauchy's Stress Principles and hence define stress vector.

(b) Establish equilibrium equations of an arbitrary volume V of continuum, subject to a system of surface forces and body forces.

(c) The stress tensor at a point P is given by

$$(\sigma_{ij}) = \begin{pmatrix} 7 & -5 & 0 \\ -5 & 3 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

Determine the stress vector on the plane passing through and parallel to the plane

$$3x_1 + 6x_2 + 2x_3 = 12$$

(c) Establish the equations of Compatibility.

(d) The Lagrangian description of deformation is given by

$$x_1 = X_1 + X_2(e^t - 1),\quad x_2 = X_1(e^t - 1) + X_2,\quad x_3 = X_3$$

Where e is a constant. Show that the Jacobian J does not vanish and determine the Eulerian equations describing the motion.

2. **Answer *any two* the following questions** **10X2=20**

(a) What is the material derivatives of a continuum? Explain in details about it.

(b) Give the Physical interpretation of Rate of Deformation tensor and Vorticity tensor.

(c) What are the Path lines and stream lines of a continuum? Give any two points of difference between path lines and stream lines.

Group-B (Lattice Theory)

1. Answer any *four* from the following questions: 4X5=20

(a) Define lattice, give an example. In lattice L, for any $a, b \in L$ prove the following
(i) $a \wedge b = a$ iff $a \le b$ and (ii) $a \vee b = b$ iff $a \le b$.

(b) Define a bounded lattice. Prove that every finite lattice L is bounded.

(c) Prove that two bounded lattice L_1 and L_2 are complemented if and only if $L_1 \times L_2$ is complemented.

(d) Prove that in any lattice distributive inequalities holds.

(e) Define atom and join-irreducible in a lattice. Prove that every atom of a lattice with zero is join-irreducible.

2. Answer any *four* from the following questions: 4X5=20

(a) Define distributive and modular lattice. Give an example of a

modular lattice which is not distributive.

b. Prove that two lattices L and M are distributive if and only if $L \times M$ is distributive.

c. A lattice L is modular if and only if for $a, b, c \in L$ the three relations,

$a \ge b,\ a \wedge c = b \wedge c,\ a \vee c = b \vee c$ imply $a = b$.

d. Prove that homomorphic image of a modular lattice is modular.

e. Let L be distributive lattice. If $a \wedge x = a \wedge y$ and $a \vee x = a \vee y$, for any element $a \in L$, then prove that $x = y$.

****