2018

MATHEMATICS

MAT 302

DYNAMICAL SYSTEMS

Full marks: 80

Time: 3 hour

The figures in the margin indicate full marks for the questions

1. Answer any *two* of the following questions: $2 \times 10 = 20$

(a) Let $F(x) = x^3 - 2$. Compute $F^2(x)$, and $F^3(x)$. Sketch the graph of $D^2(x)$ and $D^3(x)$. What will the graph of $D^n(x)$ look like?

(b) Define orbits and discuss its different types. Discuss the behavior of the resulting orbit under D for the seeds $x_0 = \frac{3}{22}, 0.3$.

(c) Give an explicit formula for $T^2(x)$ and $T^3(x)$. Also write down the general formula for $T^n(x)$.

2. Answer any *four* of the following questions: $4 \times 5 = 20$

(a) Write a short note on Graphical Analysis.

(b) Define attracting and repelling Fixed Point Theorem.

(c) Perform a complete orbit analysis for the following functions $F(x) = \frac{1}{2}x - 2$ and $F(x) = |x|$.

(d) Define unstable node, stable focus, center with diagram.

(e) For which values of λ does $F_\lambda(x) = \lambda x(1 - x)$ have a non zero attracting fixed point?

3. Answer any *two* of the following questions: $2 \times 10 = 20$

(a) Analyze the stability of Lotka-Volterra model in detail.

(b) Write about Van der Pol oscillator.

(c) Construct the phase diagram for the simple harmonic oscillator $\ddot{x} + \omega^2 x = 0$.

4. Answer any *two* of the following questions: $2 \times 10 = 20$

(a) Write in detail about the Mandelbrot Set.

(b) Give the definition of the Period-Doubling Bifurcation. Write a short note on Fractals.

(c) The function $F_\lambda(x) = x + x^2 + \lambda$ undergoes a bifurcation of fixed points at the given parameter value $\lambda = 0$ and $\lambda = -1$. Use algebraic or graphical method to identify this bifurcation as either a saddle-node or period-doubling bifurcation, or neither of these. Sketch the phase portrait for the typical parameter values below, at, and above the bifurcation value.

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