2015

MATHEMATICS

Paper : MTC 304

FUZZY SET THEORY

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer any four from the following questions : 5×4=20

(a) What is fuzzy set? Write the difference between general set and fuzzy set.

(b) What is convex fuzzy set? Prove that sum of two convex fuzzy set is convex.

(c) What is α -cut and strong α -cut? Let A be a fuzzy set defined by

$$A = \frac{0.5}{x_1} + \frac{0.4}{x_2} + \frac{0.7}{x_3} + \frac{0.8}{x_4} + \frac{1}{x_5}$$

Find all α -cuts and strong α -cuts

(d) Let A, B and C be fuzzy sets defined on the interval [0, 10] of real numbers by the membership grade functions.

$$A(x) = \frac{x}{x+2}, \quad B(x) = 2^{-x} \text{ and } C(x) = \frac{1}{1+10(x-2)^2}$$

Then find A^c, B^c and C^c.

(e) Let $f: X \to Y$ be an arbitrary crisp function. Then for any $A \in f(X)$, Prove that

$$f(A) = \bigcup_{\alpha \in [0,1]^{\alpha+}} [f(A)]$$

2. Answer any three from the following questions : $10 \times 3 = 30$

(a) Let A and B are two fuzzy numbers whose membership functions are given by

$$A(x) = \begin{cases} 0 & \text{if } x \le -1 \text{ and } 3 \le x \\ \dfrac{x+1}{2} & \text{if } -1 \le x \le 1 \\ \dfrac{3-x}{2} & \text{if } 1 < x \le 3 \end{cases}$$

and

$$B(x) = \begin{cases} 0 & \text{if } x \le 1 \text{ and } 5 < x \\ \dfrac{x-1}{2} & \text{if } 1 < x \le 3 \\ \dfrac{5-x}{2} & \text{if } 3 < x \le 5 \end{cases}$$

Then find A. B and A/B.

(b) Write the Axioms of fuzzy complement function. Give an example of fuzzy complement that satiesfies only axiomatic skeleton. Show that every fuzzy complement has at most one equilibrium.

(c) Show that the fuzzy relation R is defined by the membership matrix

$$R = \begin{pmatrix} 0.2 & 1 & 0.4 \\ 0 & 0.6 & 0.3 \\ 0 & 1 & 0.3 \end{pmatrix}$$ is max-min transitive.

(d) Let $Q = \begin{pmatrix} 0.9 & 0.6 & 1 \\ 0.8 & 0.8 & 0.5 \\ 0.6 & 0.4 & 0.6 \end{pmatrix}$ and $r = [0.6 \quad 0.6 \quad 0.5]$

Solve the fuzzy relation equation

$P \circ Q = r$, using max-min composition.

3. Answer any Six from the following questions : $5 \times 6 = 30$

(a) Let $A, B \in f(x)$ and $\alpha, \beta \in [0,1]$. prove that

if $\alpha \le \beta$ then ${}^{\beta}\beta \subseteq {}^{\alpha}A$ and ${}^{\beta+}\beta \subseteq {}^{\alpha+}A$

(b) Prove that standard fuzzy intersection is the only idempotent t-norm..

(c) Let f: $X \to Y$ be an arbitrary crisp function. Then for any $A \in f(X)$ and $\alpha \in [0, 1]$, prove with example that ${}^{\alpha+}[f(A)] \ne f({}^{\alpha+}A)$.

(d) Prove that for all $a, b \in [0, 1]$

$\max (a_1 b) \le u(a, b) \le U_{max}(a_1 b)$, Where U_{max} denotes the drastic union.

(e) What is fuzzy relation? Describe the union and intersection of fuzzy relation with example.

(f) Let A and B be two fuzzy numbers defined by

$$A = \frac{0.2}{(0,1)} + \frac{0.6}{(1,2)} + \frac{0.8}{(2,3)} + \frac{0.9}{(3,4)} + \frac{1}{4} + \frac{0.5}{(4,3)} + \frac{0.1}{(5,4)}$$

$$B = \frac{0.1}{(0,1)} + \frac{0.2}{(1,2)} + \frac{0.6}{(2,3)} + \frac{0.7}{(3,4)} + \frac{0.8}{(4,5)} + \frac{0.9}{(5,6)} + \frac{1}{6}$$

$$+ \frac{0.5}{(6,7)} + \frac{0.4}{(7,8)} + \frac{0.2}{(8,9)} + \frac{0.1}{(9,10)}$$

Then find the solution of equation $A + X = B$

(g) Let A and B be two fuzzy sets numbers defined by

$$A(x) = \begin{cases} \frac{x+2}{2}, & -2 < x \leq 0 \\ \frac{2-x}{2}, & 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

and

$$B(x) = \begin{cases} \frac{x-2}{2}, & -2 < x \leq 0 \\ \frac{6-x}{2}, & 0 < x \leq 6 \\ 0, & \text{otherwise} \end{cases}$$

Find the solution of the equation $A + X = B$

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