2018

MATHEMATICS

MAT 303

ELECTIVE I (FLUID DYNAMICS I)

Full Marks : 80

Time : 3 hrs

The figures in the margin indicate full marks for the questions

All questions are compulsory

1. Define stagnation point. Derive the equation of continuity and express it in cartesian co-ordinates. 10

2. Define viscosity. Derive the velocity of fluid at a point. Prove that at all points of the field of flow the equipotentials are cut orthogonally by the streamlines. 10

Or,

Derive the condition for any surface to be a boundary surface and deduce the equations of pathlines.

3. Derive the Lagrange's hydrodynamical equations. Mention the Euler's equation of motion in Cartesian and cylindrical coordinate system. 10

Or,

A homogeneous incompressible liquid occupies a length $2l$ of a straight tube of uniform small bore and is acted upon by a body force which is such that the fluid is attracted to a fixed point of the tube, with a force varying as the distance from the point. Discuss the motion and determine the velocity and pressure within the liquid.

4. Define sink. Prove that for irrotational incompressible two-dimensional flow $\phi(x,y)$ and $\psi(x,y)$ are harmonic functions and the family of curves $\phi =$ constant and $\psi =$ constant intersect orthogonally. 10

Or,

Define steady flow. Derive Bernoulli's equation for steady flow.

5. Define stream function and source. Give the physical interpretation of stream function. 10

Or,

What do you mean by dipole ? Derive the velocity potential of a doublet.

6. Find the complex potential for the motion due to a system consisting of a coincident line-source of strength m per unit length and line vortex of strength k per unit length in the presence of a circular cylinder of radius a, whose axis is parallel to and at a distance $b(>a)$ from the line of the source and vortex. Show that the cylinder is attracted by a force of magnitude

$$\frac{2\pi\rho a^2\left(m^2+k^2\right)}{b\left(b^2-a^2\right)}$$

per unit length. 10

Or,

Define vortex line, vortex filament, vortex tube. Prove that the complex potential of a vortex doublet is the same as that for a doublet with its axes rotated through a right angle.

7. Discuss the complex potential due to a rectilinear vortex. If n rectilinear vortices of the same strength k are symmetrically arranged along generators of a circular cylinder of radius 'a' in an infinite liquid,

prove that the vortices will move round the cylinder uniformly in time $\frac{4\pi a^2}{k(n-1)}$. 10

8. State and prove Kirchhoff vortex theorem. 10

Or,

Two point vortices each of strength K are situated at $(\pm a,0)$ and a point vortex of strength –K/2 is situated at the origin. Show that the fluid motion is stationary and find the equations of streamlines. Show that the streamlines which passes through the stagnation points meet the x-axis at $(\pm b,0)$ where, $3\sqrt{3}\left(b^3-a^3\right)^2=16a^3b$.

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