BU/EX/PG/SEM-3/303

2018

MATHEMATICS

MAT 303

ELECTIVE 1

Full Marks: 80

Time: 3 hours

The figures in the margin indicates full marks for the questions :

Attempt either Group A or Group B

Group A: Number Theory I

1. (a) Prove that $\frac{21n+4}{14n+3}$ is irreducible for every natural number n. 5

(b) Show that for any integer a, $a(a^4+4)/5$ is an integer. 5

or

Show that the product of any three consecutive integer is divisible by 3!. 5

(c) If $(a, b) = 1$, then show that for any integer c, $(ac, b) = (c, b)$. 5

(d) If q is the least positive divisor for the composite integer a, then show that $q \leqq \sqrt{a}$. 5

or

If p is a prime then show that there exists no positive integers a and b such that $a^2 = pb^2$. 5

2. (a) Show that F_5 is divisible by 641. 5

or

Prove or disprove that F_{18} is composite. 5

(b) Show that for $m \neq n, m, n \in \mathbb{N}$, $(F_m, F_n) = 1$. 5

(c) Show that the product of the first n Fermat's numbers is $2^{2^n} - 1$. 5

(d) Show that an even integer n is perfect if and only if for a prime p, $n = 2^{p-1}M(p)$, where $M(p)$ is a Mersenne prime. 5

or

Show that if $a^n - 1$, $n > 1$ is prime. Then $a = 2$ and n is prime. Hence show that if n is a composite number, then $M(n)$ is not a Mersenne prime. 4+1=5

3. (a) Let p be a prime number. Show that $2^p + 3^p$ is not a perfect power. 5

or

Show that $3^{4n+2} + 5^{2n+1} \equiv 0(\mod 14)$. 5

(b) Find the last unit digit of 3^{400}. 5

(c) If $a^{m-1} \equiv 1(\mod m)$ and $a^n \not\equiv 1(\mod m)$ for any proper divisor of $m-1$, then show that m is a prime. 5

or

Show that the equation $ax + b \equiv 0(\mod m)$ has a solution if and only if $(a, m)|b$. 5

(d) Find the least natural number which when divided by 3, 5 and 2 leaves in order the remainders 2, 3 and 5 respectively. 5

4. (a) Show that there is exactly $\frac{p-1}{2}$ quadratic residue and $\frac{p-1}{2}$ quadratic residue $\mod p$. 5

or

Show that if $(p, a) = 1$, then $x^2 \equiv a(\mod p)$ has two solutions. 5

(b) Let $(a, p) = 1$, then show either *one* of the following 5

i. If $p \equiv 1(\mod 4)$, then $-a$ is $R \mod p$ if and only if a is R.

ii. If $p \equiv 3(\mod 4)$, then $-a$ is $N \mod p$ if and only if a is R.

(c) If $P(m, n)$ be a lattice point in the first quadrant and $(m,n) = 1$, then show that there does not exists any lattice point on the line OP, excluding the points O and P. 5

(d) Show that the congruence $x^2 \equiv 15(\mod 1093)$ has no solution. 5

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