2016

MATHEMATICS

PAPER : MTC 201
COMPLEX ANALYSIS & LINEAR ALGEBRA
(Old Course)

Full Mark : 80
Time : 3 Hrs

Figures in the right hand margin indicate full marks for the question

(Use separate script for both the groups)

Group –A (*Complex Analysis*) **Marks : 50**

1\. (a) State and prove the necessary condition for a complex valued function $f(z)$ is analytic. 7

(b) Prove that $u = e^{-x}(xsiny - ycosy)$ is harmonic function 3

2\. Answer any two questions: $6 \times 2 = 12$

(a) State and prove the cauchy's inequality.

(b) If a>e, use Rouche's theorem to prove that the equation $e^z = az^n$ has n roots inside the circle $|z| < 1$.

(c) Evaluate the integral $\int_0^{2\pi} \frac{\cos 2\theta \, d\theta}{5+4\cos\theta}$.

3. Obtain the Laurent's series which represent the function $\frac{z^2-1}{(z+2)(z+3)}$ in the region 9

(i) within $|z| = 1$.

(ii) in the annular region between $|z| = 2$ and $|z| = 3$.

(iii) exterior to $|z| = 3$.

4. What kind of singularity have the following functions: $2\,1/2 \times 2 = 5$

(i) $\frac{1}{1-e^z}$ at $z = 2\pi i$. (ii) $\frac{1}{\sin z - \cos z}$ at $z = \frac{\pi}{4}$.

5. The function $f(z)$ has a double pole at $z = 0$ with residue 2, a simple pole at $z = 1$, with residue 2, is analytic at all other finite points of the plane and is bounded as $|z| \to \infty$. If $f(2) = 5$ and $f(-1) = 2$ find $f(z)$. 5

6. (a) State and prove the Schwarz's Reflection Principle. 5

(b) Let $f(z)$ be an analytic function in a region R and suppose that $f(z) = 0$, at all points on an arc PQ inside R. Prove that $f(z) = 0$ throughout R.

Group-B (Linear Algebra) **Marks: 30**

1. Answer the followings: $5 \times 3 = 15$

(a) Show that the mapping $T: V_3(\mathrm{R}) \longrightarrow V_2(\mathrm{R})$ defined

as $T(a_1, a_2, a_3) = (3a_1 - 2a_2 + a_3, a_1 - 3a_2 - 2a_3)$ is a linear transformation from $V_3(\mathrm{R})$ into $V_2(\mathrm{R})$.

(b) Apply the Gram-schmidt process to the vectors $u_1 = (1,0,1)$, $u_2 = (1,0,-1)$, $u_3 = (0,3,4)$ to obtain an orthonormal basis for $R^3(R)$ with the standard inner product.

(c) Prove that two vectors a and b in a real inner product space V(R) are orthogonal if and only if $\| a + b \|^2 = \| a \|^2 + \| b \|^2$.

2. Answer any three parts : $5 \times 3 = 15$

(a) Determine the eigen values and eigen vectors of the matrix

$$\begin{pmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{pmatrix}$$

(b) Show that the minimal polynomial of the real matrix

$$\begin{pmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{pmatrix}$$

is $(x-1)(x-2)$.

(c) Verify the Cayley-Hamilton theorem for the matrix

$$A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}.$$ Also find A^{-1}.

(d) Show that the matrix $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$, is not diagonalizable.

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