2016

MATHEMATICS

PAPER : MAT 203

MATHEMATICAL METHOD

Full Mark : 80

Time : 3 Hrs

Figures in the right hand margin indicate full marks for the question

1. Answer any two questions: $2 \times 10 = 20$

a) Solve, by using the method of successive approximations, the integral equation

$$y(x) = 1 + \lambda \int_0^1 xty(t)dt.$$

b) Solve the homogeneous Fredholm integral equation

$$y(x) = \lambda \int_0^1 e^x e^t y(t)dt$$

c) Find the resolvent kernel of the Volterra integral equation with the Kernel $K(x,t) = \frac{(2+cosx)}{(2+cost)}$.

P.T.O.

2. Answer any two questions: $2 \times 10 = 20$

a) Find the Fourier cosine transform of e^{-x^2}.

b) Find $F(x)$ if its Fourier sine transform is $\frac{s}{1+s^2}$.

c) Use finite cosine transform to solve

$$\frac{\partial V}{\partial t} = k\frac{\partial^2 V}{\partial x^2}, 0 < x < \pi, t > 0$$

with the boundary conditions $\frac{\partial V}{\partial x} = 0$ when x=0 and $x = \pi, t > 0$ and the initial condition $V = f(x)$ when $t = 0, 0 < x < \pi$.

3. Answer any four questions: $4 \times 5 = 20$

a) Evaluate $\int_0^\infty te^{-t}sint dt$.

b) If $L\{F(t)\} = f(s)$ then $L\left\{\int_0^t F(u)du\right\} = \frac{f(s)}{s}$.

c) Evaluate $L^{-1}\left\{\frac{1}{s^2(s^2+1)}\right\}$.

d) Find $L\{J_0(t)\}$ where $J_0(t)$ is the Bessel function of order zero.

e) Show that $L\left\{\frac{e^{-at}-e^{-bt}}{t}\right\} = \ln\left(\frac{s+b}{s+a}\right)$.

P.T.O.

f) Solve the initial value problem, using Laplace transform:

$y''(x) + 2y'(x) + 5y(x) = e^{-x}sinx$

when $y(0) = 0, y'(0) = 0$.

4. Answer any two questions: $2 \times 10 = 20$

a) Show that the functional $I[x(t), y(t)] = \int_0^1 \left[2x + \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right] dt$ such that $x(0) = 1,\ y(0) = 1, x(1) = 1.5, y(1) = 1$ is stationary for $x = \frac{2+t^2}{2}, y = 1$.

b) Find the extremals of the functional $I[y(x)] = \int_{-1}^0 [480y - y'''^2]dx, y(0) = y'(0) = y''(0) = 0$.

c) Show that the geodesics on a sphere of radius a are its great circles.

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