2016

MATHEMATICS

PAPER : MAT 205

CONTINUUM MECHANICS & LATTICE THEORY

Full Mark : 80
Time : 3 Hrs

Figures in the right hand margin indicate full marks for the question

Use separate answer scripts for both the groups

Group – A (Continuum Mechanics)

1. (a) Define stress tensor. Derive the equation of equilibrium and deduce the relation $\sigma_{ij,j} + \rho b_i = 0$. 1+6=7

(b) Define stress invariants. Determine the principal stress values and principal directions for the stress tensor 1+6=7

$$\sigma_{ij} = \begin{pmatrix} \tau & \tau & \tau \\ \tau & \tau & \tau \\ \tau & \tau & \tau \end{pmatrix}$$

(c) The stress tensor at a point P is given with respect to the axes $Ox_1x_2x_3$ by the values 6

$$\sigma_{ij} = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix}$$

P.T.O.

Determine the principle stress values, stress invariants and the principle directions represented by the axes $Ox_1'x_2'x_3'$.

2. (a) Define stream lines and path lines. Find the stream lines and path lines of the particles for the two dimensional velocity field

$$u=\frac{x}{1+t},\ v=y,\ w=0$$ 1+1+5=7

(b) For the motion $x_1=X_1,\ x_2=X_2+X_1\left(e^{-2t}-1\right),$ $x_3=X_3+X_1\left(e^{-3t}-1\right)$ compute the rate of deformation D and the vorticity tensor V. Compare D with $d\varepsilon_{ij}/dt$, the rate of change of the Eulerian small strain tensor E 5

(c) Assuming the constitutive equation $\sigma_{ij}=(-p+\lambda^* D_{kk})\delta_{ij}+2\mu^* D_{ij}$, show that the equation of motion is

$\rho\dot{v}_i=\rho b_i-p_{,i}+(\lambda^*+\mu^*)v_{j,ji}+\mu^* v_{i,jj}$. Deduce that

(i) $\rho\dot{v}_i=\rho b_i-p_{,i}+\mu^* v_{i,jj}$ for incompressible fluid

(ii) $\rho\dot{v}_i=\rho b_i-p_{,i}+\frac{1}{3}\mu^* v_{j,ji}+\mu^* v_{i,jj}$ for compressible fluid

the fluid motion is very slow so that the higher order terms in the velocity are negligible, show that in a steady incompressible flow with zero body force the pressure is harmonic. 2+1+2+3=8

Group – B (Lattice Theory)

1. Answer the followings : 1×4=4

(i) Give an example of a poset with greatest element and least element.

(ii) Draw the Hasse diagram for the lattice L= {1, 2, 3, 4, 6, 12} under divisibility.

(iii) Give an example to show that union of two sublattices may not be a sub lattice.

(iv) Give an example to show that a sub lattice of lattice may not be an ideal.

2. Answer any four questions : 4×4=16

(a) Show that dual of a lattice is a lattice.

(b) Prove that a sub-lattice S of a lattice L is a convex sub-lattice if and only if $\forall a,b\in S\ (a\le b)[a,b]\subseteq S$.

(c) State and prove the duality principle of a lattice.

(d) Establish that in any lattice L,

(i) $a\wedge(b\vee c)\ge(a\wedge b)\vee(a\wedge c)$

(ii) $a\vee(b\wedge c)\le(a\vee b)\wedge(a\vee c)$, where $a,b,c\in L$

(e) Show that intersection of two ideals of a lattice is again an ideal.

3. Answer any three questions : 3×5=15

(a) Show that a lattice L is a chain iff all ideals in L are prime.

(b) Prove that two bounded lattice A and B are complemented if and only if $A\times B$ is complemented.

(c) Prove that homomorphic image of a modular lattice is modular.

(d) Let I be a prime ideal of a lattice L, then show that $L-I$ is a dual prime ideal.

4. Show that every Boolean algebra is a Boolean ring with unity. 5

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