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63/2 (SEM-4) MAT 401 A,B (N/O)

2023

MATHEMATICS

Paper Code : MAT 401 (A)

(Advanced Topology)

Full Marks – 80

Pass Marks – 32

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. (a) (i) Let A be a subset of a topological space (X, T). Show that $x \in \bar{A}$ if and only if there is a net in A converges to x. 5

Or

(ii) Does the collection of all cluster point of a net in a topological space is closed ? Justify your answer. 1+4=5

[Turn over

(b) Show that a filter F on a set X is an ultra filter if and only if for each $A \subseteq X$, either $A \in F$ or $A^c \in F$. 5

(c) Does a paracompact subset of a Hausdorff space is closed ? Justify your answer. 1+4=5

(d) (i) Show that every metrizable space is paracompact. Does the result is true for compactness ? 4+1=5

Or

(ii) Show that every regular Lindelöf space is paracompact. Does the result is true for compactness ? 4+1=5

2. (a) (i) Show that every uniformity on a non-empty set X, generates a topology on X. 5

Or

(ii) Let (X, U) be an uniform space. Show that the collection of open, symmetric members of U form a base for U. 5

(b) Show that every continuous function on a fine space to some uniform space is uniformly continuous. 5

(c) Show that a uniformly continuous function on a subset A of an uniform space X to a complete uniform space Y can be extended to $\bar{A}$. 5

(d) Show that a compact Hausdorff space (X, I) admits a unique proximity, given by the elementary proximity $A \delta B$ if and only if $\bar{A} \cap \bar{B} = \Phi$, $\forall A, B \in \mathbb{P}(X)$. 5

3. (a) (i) Let $A \subseteq R$ be a discrete subgroup. Then show that $A = a\mathbb{Z}$, for an unique real number $a \geq 0$. 5

Or

(ii) Show that every proper closed subgroup of the additive group $\mathbb{R}$ is of the form $a\mathbb{Z}$, for an unique real number $a \geq 0$. 5

(b) Let H be an inert subgroup of a topological group (G, ., I). Show that $I_H := \{U \subseteq G \mid {}_gU \cap H$ is open in H for every $g \in G\}$ is a group topology on G. 5

(c) Let G be a locally compact group and H be a closed subgroup of G. Show that G/H is locally compact. 5

(d) Let X be a path connected space. Show that $\pi_1(X, x_0)$ is an isomorphic to $\pi_1(X, x_1)$, where x_0 and x_1 are two points of X. 5

Or

Prove or disprove that the fundamental group of S^1 is an isomorphic to $\mathbb{Z}$. 5

4. (a) (i) Let G and H be topological group, and $\phi: G \to H$ be a homomorphism onto H. Show that ϕ is an open function if and only if ϕ is uniformly open for the structure pair $(S_l(G), S_l(H))$. 5

Or

(ii) Let G and H be topological group, and $\phi: G \to H$ be an open continuous homomorphism onto H. Show that ϕ is uniformly open for the pairs of structures $(S_l(G), S_r(H))$ and $(S_r(G), S_l(H))$ if and only if H has equal uniform structures. 5

(b) Let G be a locally compact topological group such that every open σ-compact subgroup has equal uniformities, then show that G satisfies the G_δ condition. 5

(c) (i) Let F be an equicontinuous family of functions. Show that the pointwise closure $\overline{F}$ is also equicontinuous. 5

Or

(ii) On an equicontinuous family F, show that the compact-open topology reduces to the pointwise topology. 5

(d) Let X be a Hausdorff, or regular, k-space, and Y be a Hausdorff uniform space. Let F be a family of continuous functions from X to Y such that F is compact in the compact-open topology.

Show that F is equicontinuous on each compact subset of X. 5

Paper Code : MAT 401 (B)

Fluid Dynamics (New)

Full Marks – 60

Pass Marks – 24

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×5=5

   (a) When a fluid is said to be anisotropic fluid ?

   (b) Give one example-of surface force.

   (c) Which quantity is associated with connective derivative ?

   (d) Under which condition a fluid motion is irrotational ?

   (e) For which value of Reynold's number a flow becomes laminar ?

2. Answer the following questions : 2×5=10

   (a) Write the equations of pathlines and streamlines.

   (b) State two differences between source and sink.

   (c) What is vortex line and vortex tube ?

   (d) Find the complex potential for the flow $w = ikz$.

   (e) What are progressive and stationary waves ?

3. Answer any *five* from the following questions : 5×5=25

   (a) Determine the acceleration at the point (2,1,3) at t = 0.5 sec, if $u = yz + t$, $v = xz - t$, and $w = xy$.

   (b) Determine the Euler's equation of motion in Cartesian coordinate system.

   (c) Derive the equation $\frac{p}{\rho}+\frac{1}{2}q^2+\Omega=$ constant.

   Where the symbols have their usual meanings.

   (d) Derive the relation between rectangular components of stress in a fluid.

   (e) Determine the complex potential due to a doublet.

   (f) Discuss the dynamical significance of group velocity.

   (g) Discuss Couette flow between two parallel plates.

4. Answer any *two* from the following questions : 10×2=20

(a) Derive the Navier-Stokes equations of motion for a vicscous compressible fluid in Cartesian coordinates.

(b) State and prove Buckingham's Pi Theorem.

(c) Prove that the velocity of propagation of a wave $\eta = a\sin(mx - nt)$ at the surface of water of uniform depth is $C^2 = \frac{g}{m}\tan h\, mh$.

(Theory Paper)

Paper Code : MAT 401 (B)

Fluid Dynamics (Old)

Full Marks – 80

Pass Marks – 32

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×6=6

(a) What do you mean by ideal fluids ?

(b) When a flow is said to be steady ?

(c) Are streamlines real or imagined ?

(d) State Blasius's theorem.

(e) What is an equation of pathline ?

(f) For which value of Reynold's number a fluid flow is laminar ?

2. Write short notes on the following : 5×2=10

(a) Uniform flows

(b) Stress of a fluid

(c) Dynamical similarity

(d) Vortex motion

(e) Source.

3. Answer any *six* from the following : 5×6=30

(a) Show that the polar form of an equation of continuity for a two dimensional incompressible fluid is $\frac{\partial}{\partial r}(ru)+\frac{\partial v}{\partial \theta}=0$.

(b) Derive equation of continuity in Cylindrical coordinate system.

(c) Discuss Euler's equation of motion by vector method in fluid dynamics.

(d) Derive Bernoulli's equation for steady flow.

(e) Discuss the flow due to an uniform line doublet at origin of strength m per unit length and its axis being along the x-axis.

(f) State and prove Kirchhoff vortex theorem.

(g) Discuss plane Poiseuille flow for parallel plates.

(h) Prove that the difference of the values of a stream function at any two points represents the flow across that curve, joining the two points.

(i) Prove that the group velocity for shallow water is equal to the wave velocity.

4. Answer any *two* from the following questions : 10×2=20

(a) Derive the velocity of fluid at a point. Prove that at all points of the field of flow the equipotentials are cut orthogonally by the streamlines.

(b) Define stream function. Give the physical interpretation of stream function.

(c) Discuss the complex potential due to a rectilinear vortex. If n rectilinear vortices of the same strength k are symmetrically arranged along generators of a circular cylinder of radius 'a' in an infinite liquid, prove that the vortices will move round the cylinder uniformly in time $\frac{4\pi a^2}{k(n-1)}$.

5. Answer any *one* question : 14×1=14

(a) A long straight pipe of length L has a slowly tapering circular cross-section. It is inclined so that its axis makes and angle α to the horizontal with its smaller cross-section downwards. The radius of the pipe at its upper end is twice that of at its lower end and water is pumped at a steady rate through the pipe to emerge at atmospheric pressure. If the pumping pressure is twice the atmospheric pressure, show that the fluid leaves the pipe with a speed U given by

$$U^2 = \frac{32}{15}\left[gL\sin\alpha + \frac{\Pi}{\rho}\right].$$

Where Π is an atmospheric pressure.

(b) Discuss the complex potential due to a rectilinear vortex. Two point vortices each of strength K are situated at $(\pm a, 0)$ and a point vortex of strength –K/2 is situated at the origin. Show that the fluid motion is stationary and find the equations of streamlines. Show that the streamlines which pass through the stagnation points meet the x-axis at $(\pm b, 0)$ where,

$$3\sqrt{3}\left(b^3 - a^3\right)^2 = 16a^3b.$$