Total No. of printed pages = 24

63/2 (SEM-4) MAT 403 (A,B,C)

2023

MATHEMATICS

(Theory Paper)

Paper Code : MAT 403 (A)

(Fuzzy Logic and Fuzzy Control System)

Full Marks – 80

Pass Marks – 32

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. Answer any *four* from the following questions : $4\times5=20$

(a) What is Fuzzy Logic ? Write the differences between Fuzzy Logic and Fuzzy Proposition.

(b) Write about unconditional and qualified proposition with example.

(c) Explain Fuzzy Quantifiers of second kind with example.

[Turn over

(d) Let $A = \frac{0.1}{x_1} + \frac{0.8}{x_2} + \frac{1}{x_3}$ and $B = \frac{0.5}{y_1} + \frac{1}{y_2}$ then determine R (X, Y).

(e) Discuss about the Composition Rule of inference.

2. Answer any *two* from the following questions : 10×2=20

(a) What is Fuzzy implications ? Write the reasonable axioms of Fuzzy Implications.

(b) Write briefly with diagram about the Fuzzy Expert system.

(c) Write the general schema form of multiconditional approximate reasoning. Write the steps of determine B as a method of interpolation.

3. Answer any *two* from the following questions : 10×2=20

(a) Write all overview of Fuzzy controllers.

(b) Write on S-implication and R-implication.

(c) Write the steps to discuss the main issues involve in the design of Fuzzy controller for stabilizing an inverted pendulum.

4. Answer any *two* from the following questions : 10×2=20

(a) Write on Multi-person decision making.

(b) Explain on individual decision making with example.

(c) Solve the following Fuzzy linear Programming problem

$$\max Z = 0.5x_1 + 0.2x_2$$

such that

$$x_1 + x_2 \le B_1$$
$$2x_1 + x_2 \le B_2$$
$$x_1, x_2 \ge 0$$

where

$$B_1(x) = \begin{cases} 1 \,; \text{for } x \le 300 \\ \frac{400}{100} \text{ for } 400 \le x \le 400 \\ 0 \text{ for } x > 400 \end{cases}$$

and

$$B_2(x) = \begin{cases} 1 \,; \text{for } x \le 400 \\ \frac{500}{100} \text{ for } 400 \le x \le 500 \\ 0 \text{ for } x > 500. \end{cases}$$

(Theory Paper)

Paper Code : MAT 403 (B)

(Relativity and Cosmology)

Full Marks – 80

Pass Marks – 32

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : $1\times6=6$

   (a) What do you mean by metric or line element ?

   (b) Which phenomenon is explained by geodesic principle ?

   (c) Which fundamental physical theory is based on principle of covariance ?

   (d) What is the phenomenon of bending of light rays called ?

   (e) What is the current estimate of the curvature of the universe ?

   (f) What is the physical significance of the Ricci tensor in the Einstein Field equation ?

2. Write short notes on the following : $2\times5=10$

   (a) Closed universe

   (b) Gravitational redshift

   (c) Advance of perihelion

   (d) Hubble's law

   (e) Age of the universe.

3. Answer any *six* from the following questions : $6\times5=30$

   (a) What is Weyl's postulate, and how does it relate to the Theory of General relativity ? Discuss some of the key assumptions and implications of the postulate, including its role in defining the geometry of space-time.

   (b) Discuss the formula for energy momentum tensor for a perfect fluid in the form

   $$T_{\nu}^{\mu}=(\rho+p)v_{\mu}v^{\nu}-pg_{\mu}^{\nu}.$$

   (c) Discuss the motion of free particle in case of weak static field.

   (d) What is the Einstein Field equation with cosmological term, and how does it differ from the original Einstein Field equation ? Discuss some of the implications of adding a cosmological term to the equation.

(e) Write a brief note about the matter dominated era of the universe.

(f) What is variational principle ? Derive the field equation in empty space from variational principle.

(g) Prove that Einstein universe is not an Einstein space whereas de-Sitter's universe is Einstein space.

(h) How does critical density of the universe relate to the overall geometry of space-time ? Discuss the implications of a universe with a density higher or lower than the critical density.

(i) The mass of the sun is 2×10^{23}gm, radius of the sun is 6.96×10^{10} cm and the value of the gravitational constant is 6.67×10^{-8} cm^3/g. By what angle would a light grazing the sun bend ?

4. Answer any *two* of the following questions : $10\times2=20$

(a) Explain the equivalence principle in general relativity, and discuss its implications for the behavior of matter and energy in gravitational fields. How does the equivalence principle relate to the concept of curved space-time ? $5+5=10$

(b) What exactly is Einstein's and de-Sitter's Universe ? Compare them with the actual universe. $2+8=10$

(c) Derive the Freidmann-Robertson-Walker (FRW) metric. 10

5. Answer any *one* of the following questions : $14\times1=14$

(a) Discuss the three crucial tests of general relativity.

(b) Derive Schwarzschild's interior solution of a spherically symmetric distribution of matter with constant density.

(Theory Paper)

Paper Code : MAT 403 (C) (New)

(Operations Research)

Full Marks – 60

Pass Marks – 24

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed : 1×4=4

   (a) State any one advantage of simplex method of solving an LPP over graphical method.

   (b) Which method will be used to find an optimal solution when there are four decision variables in an LPP ?

   (c) What is the difference between feasible solution and basic feasible solution ?

   (d) Define a saddle point in a game.

2. Answer the following questions in brief : 2×4=8

   (a) What are slack and surplus variables ?

   (b) Define State-transition matrix.

   (c) Explain M/M/l model of Queuing theory.

   (d) What are the basic characteristics of a Queuing system ?

3. Answer any *two* of the following : 6×2=12

   (a) "OR is an aid for the executive in making his decision by providing him with the needed quantitative information, based on the scientific method analysis." Discuss the statement in detail, illustrating it with OR methods that you know.

   (b) A company has three operational departments (weaving, processing and packing) with capacity to produce three different types of clothes namely suitings, shirtings and woollens yielding a profit of Rs. 2, Rs. 4 and Rs. 3 per meter respectively. One meter of suiting requires 3 minutes in weaving, 2 minutes in processing and 1 minute in packing. Similarly, one meter of shirting requires 4 minutes in weaving, 1 minute in processing and 3 minutes in packing, while one meter woollen requires 3 minutes in each department. In a week, total run time of each department is 60,40 and 80 hours for weaving, processing and packing respectively.

Formulate the Linear Programming Problem to find the product mix to maximize the profit.

(c) Solve the following LPP by graphical method:

Minimize $z = 5x_1 + 2x_2$
subject to the constraints

$$x_1 + x_2 \le 2$$
$$3x_1 + 3x_2 \ge 12$$
$$\text{and } x_1, x_2 \ge 0$$

4. Answer any *two* of the following questions: 4×2=8

(a) Discuss the arrival and service process of waiting line model. Write the standard method of expressing the queuing problem.

(b) Find the expressions for the following in the queuing model (M/M/1) : (∞/FIFO):

(i) the expected (average) number of customers in the queue (L_q)

(ii) the expected (average) number of customers in nonempty queues (L_n).

(c) Arrivals at a telephone both are considered to be Poisson at an average time of 8 min between our arrival and the next. The length of the phone call is distributed exponentially, with a mean of 4 min.

Determine the following:

(i) Expected fraction of the day that the phone will be in use.

(ii) Expected number of units in the queue

(iii) Expected number of units in the system.

(iv) Expected waiting time in the system.

5. Answer any *two* of the following questions: 6×2=12

(a) Show that a game can be formulated as an LPP.

(b) Solve by using dominance property, the following game:

		B		
		I	II	III
A	I	1	7	2
	II	6	2	7
	III	6	1	6

(c) State Maximin-Minimax Principle. Show that for any (2×2) two person zero-sum game without any saddle point having the payoff matrix for player A,

$$\begin{array}{c c} & \begin{array}{cc} B_1 & B_2 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \end{array} & \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \end{array}$$

the optimum mixed strategies

$$S_A = \begin{bmatrix} A_1 & A_2 \\ p_1 & p_2 \end{bmatrix} \text{ and } S_B = \begin{bmatrix} B_1 & B_2 \\ q_1 & q_2 \end{bmatrix}.$$

are given by $\frac{p_1}{p_2} = \frac{a_{22} - a_{21}}{a_{11} - a_{12}}, \frac{q_1}{q_2} = \frac{a_{22} - a_{12}}{a_{11} - a_{21}}$,

where $p_1 + p_2 = 1$. Also show that value of game V to A is given by

$$V = \frac{a_{11}a_{12} - a_{21}a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

6. Answer any *two* of the following questions : 4×2=8

(a) Give the procedure for determining steady-state condition.

(b) The 'School of International Studies for Population' found out by its survey that the mobility of the population (in per cent) of a State to a Village, Town and City is in the following percentages :

		To		
		Village	Town	City
Form	Village	50	30	20
	Town	10	70	20
	City	10	40	50

What will be the proportion of population in village, town and city after two years, given that the present population has proportions of 0.7, 0.2 and 0.1 in the village, town and city respectively ?

(c) Consider the Markov chain with three states, $S = \{1, 2, 3\}$, that has the following transition matrix :

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{3} & 0 & \frac{2}{3} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

Draw the state transition diagram for this chain.

If we know $P(X_1 = 1) = P(X_1 = 2) = \frac{1}{4}$, find $P(X_1 = 3, X_2 = 2, X_3 = 1)$. 2+1+1=4

7. Answer either (a) or (b) and (c) : 8

(a) Write the Kuhn-Tucker conditions for the problem :

$$\text{Max, } Z = -x_1^2 - x_2^2 - x_3^2 + 4x_1 + 6x_2$$

Subject to : $x_1 + x_2 \leq 2$

$2x_1 + 3x_2 \leq 12$

and $x_1, x_2 \geq 0$

(b) Solve the following Non-linear Programming Problem :

$$\text{Max } Z = 4x_1 - x_1^2 + 8x_2 - x_2^2$$

subject to the constraints

$x_1 + x_2 = 12$

$x_1, x_2 \geq 0.$ 4

(c) Solve the following Non-linear Programming Problem using the Lagrangian multipliers :

Optimize

$$Z = 2x_1^2 + x_2^2 + 3x_3^2 + 10x_1 + 8x_2 + 6x_3 - 100$$

subject to the constraints

$x_1 + x_2 + x_3 = 20$

$x_1, x_2, x_3 \geq 0$ 4

(Theory Paper)

Paper Code : MAT 403 (C) (Old)

(Operations Research)

Full Marks – 80

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Draw the state transition diagram for this chain.

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Max, $Z = -x_1^2 - x_2^2 - x_3^2 + 4x_1 + 6x_2$

Subject to : $x_1 + x_2 \leq 2$

$2x_1 + 3x_2 \leq 12$

and $x_1, x_2 \geq 0$

(b) Solve the following Non-Linear Programming Problem :

Max $Z = 4x_1 - x_1^2 + 8x_2 - x_2^2$

subject to the constraints

$x_1 + x_2 = 2$

$x_1, x_2 \geq 0$ 4

(c) Solve the following non-linear programming problem using the Lagrangian multipliers :

Optimize

$Z = 2x_1^2 + x_2^2 + 3x_3^2 + 10x_1 + 8x_2 + 6x_3 - 100$

subject to the constraints

$x_1 + x_2 + x_3 = 20$

$x_1, x_2, x_3 \geq 0$ 4

8. Answer any *two* of the following questions : 10×2=20

(a) Solve the following LPP by Simplex method :

Maximize $Z = 5x_1 + 2x_2 + 3x_3 - x_4 + x_5$

Subject to $x_1 + 2x_2 + 2x_3 + x_4 = 8$

$3x_1 + 4x_2 + x_3 + x_5 = 7$

$x_1, x_2, x_3, x_5 \geq 0$

(b) Using Simplex method, solve the following LPP :

Max $Z = 16x_1 + 17x_2 + 10x_3$

Subject to the constraints

$$x_1 + x_2 + 4x_3 \le 2000$$
$$2x_1 + x_2 + x_3 \le 3600$$
$$x_1 + 2x_2 + 2x_3 \le 2400$$
$$x_1 \le 30$$
$$x_1, x_2, x_3 \ge 0.$$

(c) Solve the following game by Simplex method. Find the optimal strategy and value of the game.

B

	1	–1	3
A	3	5	–3
	6	2	–2