63/2 (SEM–4) PHY 401

2024

PHYSICS

Paper : PHY 401

(Statistical Mechanics)

Full Marks : 80

Pass Marks : 32

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following : 1×5=5

   (a) Bosons and fermions are different, because bosons do not obey

   (i) Heisenberg uncertainly principle

   (ii) Pauli exclusion principle

   (iii) Hund's rule

   (iv) None of the above

(b) A macroscopic system where energy, volume and particle exchange can take place is called

(i) isolated open system

(ii) closed system

(iii) open system only

(iv) universe

(c) Symmetry of wave function associated with a system of particles in quantum statistics is determined by the

(i) mass of the particle

(ii) charge on the particle

(iii) spin of the particle

(iv) energy of the particle

(d) The function of Fermi-Dirac distribution mainly deals with

(i) range of electrons

(ii) available number of electrons

(iii) chances of electrons levels occupancy

(iv) types of materials

(e) In a microcanonical ensemble, a system *A* of fixed volume is in contact with a large reservoir *B*. Then

(i) *A* can exchange neither energy nor particle with *B*

(ii) *A* can exchange both energy and particles with *B*

(iii) *A* can exchange only energy with *B*

(iv) *A* can exchange only particles with *B*

2. Answer the following : ***(any five)*** $2\times5=10$

(a) Distinguish between microcanonical and canonical ensemble.

(b) Discuss Gibbs paradox in statistical mechanics.

(c) Discuss the conditions for a gas of fermions to be in degenerate state.

(d) What is the significance of thermal de Broglie wavelength ?

(e) What is the significance of Fermi Temperature (T_F) for a Fermion gas ?

(f) Define partition function in statistical mechanics.

3. Answer the following : ***(any five)*** 5×5=25

(a) Use quantum counting of microstate to show that it is same as the number of microstate for a single particle in Γ-space. Show that the expression for density of states is

$$g(E,V)=\frac{d\Omega}{dE}=\frac{V}{h^3}\frac{\pi}{4}(8m)^{3/2}E^{1/2}$$

(b) Derive the relation $S = K_B \ln \Omega(N,V,E)$, where S defines the entropy of the macroscopic system and $\Omega(N,V,E)$ represents the number of microstates corresponding to thermodynamics parameters (N, V, E).

Or

Copper contains $8{\cdot}5\times10^{28}$ free electrons per cubic meter. Assuming that each copper atom donates one free electron, calculate the Fermi energy for these electrons and also calculate the number of quantum states in the energy range $E_F \pm K_B T$ for the free electrons at 300 K in a volume 1 meter cube.

(c) For a canonical ensemble, derive the expression for probability distribution and then for the canonical partition function.

(d) Using grand canonical partition function derive the expression for Fermi-Dirac distribution function ($\langle n_s \rangle$, the average number of particles in s-th state) and discuss briefly the characteristics of fermion gas obeying FD statistics.

(e) Show that under certain limit both the Bose-Einstein and Fermi-Dirac statistics lead to the Maxwell-Boltzmann distribution (classical statistics).

(f) Estimate the Fermi energy, Fermi temperature and Fermi pressure of silver atom (atomic weight 107·87 and density 10·5 gm/cc). Assume that one silver atom donates one conduction electron.

4. Answer the following : ***(any four)*** 10×4=40

(a) Considering an ideal gas in microcanonical ensemble deduce the expression for number of microstates. Also derive the expression for entropy and the equation of state. 5+5=10

(b) Define Bose-Einstein condensation. Discuss how a boson gas undergoes phase transition and show how the heat capacity (C_V) and pressure (P) of a boson gas are different from ideal Maxwell-Boltzmann gas (classical gas) at the condensation temperature. 2+4+4=10

(c) Derive the expression for energy density (energy spectrum) of radiation from a blackbody. Show that emissivity of the radiation is proportional to the fourth power of the temperature. 5+5=10

(d) What is fluctuation in thermodynamic quantities? Discuss how Einstein developed the theory of Brownian motion using Gaussian distributed Brownian particles. 2+8=10

(e) Discuss Fermi-Dirac distribution around absolute zero ($T = 0$) temperature and define Fermi temperature. Use FD statistics to explain *any one* of the *two* physical phenomena —

(i) thermionic emission ;

(ii) magnetic susceptibility of an ideal Fermi gas. 5+5=10

(f) State the characteristics of a white dwarf. Why the fermion (electron) gas at the interior of a white dwarf is in highly generate state? Calculate the electron degeneracy pressure inside a white dwarf. Show that for extreme relativistic case balancing with gravitational pressure of the star leads to the Chandrasekhar's limit. 2+2+3+3=10
