2023

ECONOMICS

Paper : ECOHC3066

(Mathematical Methods for Economics-II)

Full Marks : 80
Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct option from the following (any *six*) : 1×6=6

(a) The convexity of the indifference curve requires

(i) $\frac{dy}{dx} \leq 0$ and $\frac{d^2y}{dx^2} > 0$

(ii) $\frac{dy}{dx} \geq 0$ and $\frac{d^2y}{dx^2} < 0$

(iii) $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} > 0$

(iv) $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} < 0$

(b) Which of the following is a linear equation?

(i) $2x+5=0$

(ii) $3x^2+7x+2=0$

(iii) $6x^3+8x^2+2x+9=0$

(iv) None of the above

(c) Identify the orthogonal matrix.

(i) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(ii) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(iv) None of the above

(d) If $\frac{dy}{dx}=2x+y$, then the order and degree of the differential equation are respectively

(i) first-order and fourth-degree

(ii) second-order and first-degree

(iii) first-order and first-degree

(iv) third-order and fifth-degree

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(e) Homogeneous differential equation is

(i) a differential equation in which the RHS is zero

(ii) a differential equation where RHS is non-zero

(iii) a differential equation with non-linear terms

(iv) a differential equation having linear terms

(f) $\frac{dy}{dx}+5y=0$ is

(i) a homogeneous differential equation

(ii) a non-homogeneous differential equation

(iii) a non-linear differential equation

(iv) an inequality differential equation

(g) If $A=\begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix}$, what is the trace of A (tr A)?

(i) 3

(ii) 4

(iii) 8

(iv) None of the above

(Turn Over)

(h) A consumer has a utility function $U = U(x) = ax^b$. His marginal utility function is given by

(i) abx^{b-1} (ii) abx^{a-1}

(iii) bx^{b-1} (iv) ax^{a-1}

(i) An equation is said to be quadratic equation if the highest power of the unknown variable is

(i) 1 (ii) 2

(iii) 3 (iv) 4

(j) If $y = \sqrt{x}$, then $\frac{dy}{dx}$ is equal to

(i) $\frac{1}{2\sqrt{x}}$ (ii) $\frac{1}{\sqrt{x}}$

(iii) $\frac{1}{\sqrt{2}}$ (iv) None of the above

2. Answer the following questions (any *five*) : 2×5=10

(a) Obtain the general solution of the following :

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} + 12y = 0$$

(b) A utility function is given by $U = x^3 - 5xy^2 + y^3$, then find the marginal utilities of x and y.

(c) Define the Hessian matrix with an example.

(d) What are the first and second order conditions for maxima and minima?

(e) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 9 & 10 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 9 & 10 \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$; find $A + 2B$.

(f) Can we add 2×2 matrix with 2×3 matrix? Justify your answer.

(g) If the total cost function of a firm is given by $C = 80x - 15x^2 + x^3$, find *(i)* marginal cost function and *(ii)* average cost function.

3. Answer the following questions (any *six*) : 5×6=30

(a) Find x_1 and x_2 using Cramer's rule from the following set of simultaneous equations :

$$5x_1 + 0 \cdot 4x_2 = 12$$
$$3x_1 + 3x_2 = 21$$

(b) Write down the properties of a determinant.

(c) For any scalar λ and matrices A and B, prove that $\lambda A + \lambda B = \lambda(A + B)$.

(d) In a perfectly competitive market, the total revenue and total cost functions of a firm are given by

$$TR = 20Q$$

$$TC = Q^2 + 4Q + 20$$

Obtain profit maximizing output Q.

(e) Show that (i) $y = x^2$ is concave upward, and (ii) $y = 4 - 2x - x^2$ is concave downward.

(f) Find the inverse of

$$A = \begin{bmatrix} 4 & 1 & -5 \\ -2 & 3 & 1 \\ 3 & -1 & 4 \end{bmatrix}$$

(g) What is rank of a matrix? Determine the rank of the following matrix :

$$A = \begin{bmatrix} -3 & 6 & 2 \\ 1 & 5 & 4 \\ 4 & -8 & 2 \end{bmatrix}$$

(h) Make a graphical representation of the function $F(x) = 2 + 2x$.

(i) A firm produces x tonnes of output at a total cost

$$C = \frac{1}{5}x^3 - 10x^2 + 5x + 10$$

Find the level of output at which MC and AVC are minimum?

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(j) Find $\frac{dy}{dx}$ of the following implicit function :

$$x^2 + y^2 + 3x = 4y$$

4. Answer the following questions (any *two*) : 10×2=20

(a) The total cost function of a firm is

$$C = \frac{1}{3}x^3 - 5x^2 + 28x + 10$$

A tax of $ 2 per unit is levied, and the producer adds this tax to his cost. If the demand function is $P = 2500 - 5x$, where P is the price per unit, then find the profit maximizing output and the price.

(b) The equilibrium condition for three related markets is given by

$$11P_1 - P_2 - P_3 = 31$$

$$-P_1 + 6P_2 - 2P_3 = 26$$

$$-P_1 - 2P_2 + 7P_3 = 24$$

Using matrix inversion, find the equilibrium price for each market.

(c) A firm faces the production function $Q = 20K^{0.4}L^{0.6}$. It can buy inputs K and L for $ 400 a unit and $ 200 a unit respectively. What combination of L and

(Turn Over)

K should be used to maximize output if its input budget is constrained to \$ 6,000?

(d) The production function of a firm of a particular commodity is $Q = L^{0.64}K^{0.36}$. Show that the isoquant is negatively sloped and convex to the origin.

5. Answer the following questions (any *one*) : 14

(a) Given the utility function, $U = x^2 + y^2 + w^2$ subject to the linear constraint, $y + x + w = 1$, find at what point, U has a maximum or minimum value. Also determine the value of U.

(b) Solve the following national income model :

$$Y = C + 1000 + 1500$$
$$C = 40 + 0 \cdot 7\,(Y - T)$$
$$T = 100 + 0 \cdot 5Y$$

where Y, C and T represent national income, consumption and tax using Cramer's Rule.

(c) Solve the following system of equations by using matrix inversion :

$$x + y + z = 4$$
$$2x - y + 3z = 1$$
$$3x + 2y - z = 1$$

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