2018
MCA
MCA 1.3
MATHEMATICAL FOUNDATION OF COMPUTER SCIENCE

Full Marks : 75
Time : 3 Hours

The figures in the margin indicate full marks for the questions

1. **Answer the following question:** $1 \times 5 = 5$

(a) Which of the following two sets are equal?

i) $A = \{1, 2\}$ and $B = \{1\}$

ii) $A = \{1, 2\}$ and $B = \{1, 2, 3\}$

iii) $A = \{1, 2, 3\}$ and $B = \{2, 1, 3\}$

iv) $A = \{1, 2, 4\}$ and $B = \{1, 2, 3\}$

(b) A function is said to be ___________ if and only if f(a) = f(b) implies that a = b for all a and b in the domain of f.

i) One-to-many

ii) One-to-one

iii) Many-to-many

iv) Many-to-one

(c) The truth table for $(p \vee q) \vee (p \wedge r)$ is the same as the truth table for

i) $(p \vee q) \wedge (p \vee r)$

ii) $(p \vee q) \wedge r$

iii) $(p \vee q) \wedge (p \wedge r)$

iv) $(p \vee q)$

(d) Let $A = \{1, 3, 5\}$ and $B = \{3, 4, 5\}$ be sets. Which of the following are elements of $A \times B$?

i) $\{1, 3\}$

ii) $(1, 3)$

iii) (4, 5)
iv) (5, 5)

(e) Ordered collection of objects is :
i) Relation
ii) Set
iii) Function
iv) Proposition

2. **Answer the following questions:(any five)** $4 \times 5 = 20$

(a) Construct the truth table for, $(P \to Q) \wedge (Q \to R)$ and $(\neg(P \vee Q) \vee (\neg P) \wedge Q$.

(b) Show that $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \equiv R$.

(c) Obtain the disjunctive normal form of $P \leftrightarrows (\neg P \vee \neg Q)$.

(d) Test the validity of the following argument:
If milk is black then every cow is white. If every cow is white then it has four legs. If every cow has four legs then every buffalo is white and brish. The milk is black. Therefore the buffalo is white.

(e) Obtain the conjunctive normal form of $\neg(P \vee Q) \leftrightarrows (P \wedge Q)$.

(f) Test the validity of the following argument:
All integers are irrational numbers.
Some integers are powers of 2.
Therefore, some irrational number is a power of 2.

3. **Answer the following questions (any five):** $4 \times 5 = 20$

(a) Prove that the relation R in the set Z of integers defined by $aRb \Leftrightarrow (a-b)$ is even, is an equivalence relation.

(b) If R is an equivalence in a set X , then prove that R^{-1} is also an equivalence relation in X.

(c) Let R be an equivalence relation defined on a set A. Let a and b be arbitrary elements in A. Then show that if

$$[a] = [b] \Leftrightarrow (a, b) \in R \text{ if and only if } aRb.$$

(d) Let $f: R \to R$ be defined by $f(x) = 3x + 4, \forall x \in R$. Show that f is one-one, onto. Also give formula for f^{-1}.

(e) If $f: A \to B, g: B \to C$ & $h: C \to D$ are three functions such that $h_0(g_0 f)$ & $(h_0 g)_0 f$, then show that $h_0(g_0 f) = (h_0 g)_0 f$.

(f) Find the inverse of $A = \begin{bmatrix} 7 & -3 & 3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

4 **Answer the following questions (any six)** $5 \times 6 = 30$

(a) Solve $a_n - 6a_{n-1} + 8a_{n-2} = 3^n$, where $a_0 = 3$ & $a_1 = 7$.

(b) Show that a non-empty set H of a group G is a sub-group of the group G if and only if $xy^{-1} \in H, \forall x, y \in H$.

(c) Show that the set $Q - \{1\}$ of rational numbers other than 1 is an abelian group under the composition * defined as
$x * y = x + y - xy$.

(d) Show that the orders of the elements a & $x^{-1}ax$ are the same where a,,x are any two elements of a group G.

(e) Show that a sub-group H of a group G is a normal sub-group if and only if $ghg^{-1} \in H, \ \forall g \in g, \ \forall h \in H$.

(f) Prove that a group is abelian if and only if

$$(ab)^{-1} = a^{-1}b^{-1}, \quad \forall a, b \in G.$$

(g) Solve the difference equation $a_{r+1} - 3a_r = 2^r \cos r\frac{\pi}{2}$.

(h) Show that the order of every element of a finite group is finite and less than or equal to the order of the group. And also show that the order of an element a of group G is the same of its inverse a^{-1}.

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