2018

MCA

MCA : 1.5

PROBABILITY AND STATISTICS

Full Marks : 75

Time : 3 Hours

The figures in the margin indicates full marks for the questions

1. Answer the following questions: $1\times10=10$

(a) In a throw of coin what is the probability of getting head.

i) 1

ii) 2

iii) ½

iv) 0

b) In random experiment, observations of random variable are classiified as

i) Events

ii) Composition

iii) Trials

iv) Functions

(c) When a single die is rolled, what is the probability getting a prime number or a number less than 4?

i) $\frac{5}{6}$ ii) $\frac{2}{3}$

iii) $\frac{1}{2}$ iv) $\frac{1}{6}$

(d) Which of the following events are dependent?

i) Tossing a coin and selecting a cadr from a deck.

ii) Tossing a coin then tossing a second coin.

iii) Running a race and getting tired.

iv) Drawing a card from a deck and replacing it, then drawing a second card.

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(e) The measure of location which is the most likely to be influenced by extreme values in the data set is the

i) Range
ii) Median
iii) Mode
iv) Mean

(f) A numerical description of the outcome of an experiment is called a

i) Descriptive statistic
ii) Probability function
iii) Variance
iv) Random variable

(g) For a continuous random variable x, the probability density function f(x) represents

i) The probability at a given value of x.
ii) The area under the curve at x.
iii) The area under the curve to the right of x.
iv) The height of the function at x.

(h) Two events, A and B, are mutually exclusive and each have a nonzero probability. If event A is known to occur, the probability of the occurrence of event B is

i) One.
ii) Any positive value.
iii) Zero.
iv) Any value between 0 to 1.

(i) The sum of values divided by their number is called

i) Median
ii) Harmonic Mean
iii) Mean
iv) Mode

(j) The combined arithmetic mean is calculated a

i) $\frac{\overline{X_1}+\overline{X_2}}{n_1+n_2}$

ii) $\frac{n_1+n_2}{2}$

iii) $\frac{n_1\overline{X_1}+n_2\overline{X_2}}{n_1+n_2}$

iv) $\frac{n_1X_1+n_2X_2}{n_1+n_2}$

2. Answer the following questions (any five): $4\times5=20$

(a) For any two events A and B show that $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

(b) Consider the following events for a family with children:
A={Children of both sexes}, B={at most one boy}.
i) Show that A and B are independent events is a family has 3 children.
ii) Show that A and B are dependent events is a family has only 2 children.

(c) The standard deviation of two sets containing n_1 & n_2 members are σ_1 & σ_2 respectively, being measured from their respective means m_1 & m_2. If the two sets are grouped together as one set of (n_1+n_2) members, show that the standard deviation σ, of this set, measured from its mean is given by,

$$\sigma^2=\frac{n_1\sigma_1{}^2+n_2\sigma_2{}^2}{n_1+n_2}+\frac{n_1n_2}{(n_1+n_2)^2}\times(m_1+m_2)^2$$

(d) Five card are numbered 1 to5. Two cards are drawn at random(without replacement) to yield the following equiprobable space S with C(5,2)=10 elements:
S={ {1,2},{1,3},{1,4},{1,5},{2,3},{2,4},{2,5},{3,4},{3,5},{4,5}},
Let X denote the sum of the numbers drawn.
i) Find the distribution f of X.
ii) Find E(X).

(e) Let A and B be events with $P(A) = 0.6, P(B) = 0.3$ and $P(A \cap B) = 0.2$ find $P(A|B), P(B|A), P(A \cup B), P(A^c)$ & $P(B^c)$.

(f) Suppose a student dormitory in a college consists of:

i) 40 percent freshmen of whom 15 percent are New York residents.

ii) 25 percent sophomores of whom 40 percent are New York residents.

iii) 20 percent juniors of whom 25 percent are New York residents.

iv) 15 percent seniors of whom 20 percent are New York residents.

A student is rendomly selected from dormitory.

a) Find the probability that the student is a New York resident.

b) If the student is a New York resident, find the probability that the student is a, i) freshman, ii) junior.

(g) Show that for any discrete diatribution, the standard deviation is not less than the mean deviation from the mean.

3. Find the standard deviation, Q_1, Q_3 and co-efficient of variation of the following values of the following table giving wages of 230 persons: 10

Wages in Rs.	140-160	160-180	180-200	200-220	220-240	240-260	260-280	280-300
No. Of persons	12	18	35	42	50	45	20	8

4. A fair coin is tossed 3 times yielding the following 8-element equiprobable space: 10

$S=\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$ (Thus each point in S occurs with probability $\frac{1}{8}$). Let X equal to zero or one accordingly as a head or a tail occurs on the first toss, and let Y equal the total number of heads that occurs.

a) Find the distribution g of X and the distribution h of Y.

b) Find joint distribution of X and Y

c) Find Cov(X,Y), the covariance of X and Y.

d) Find $\rho(X,Y)$ the correlation of X and Y.

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5. The following marks have ben obtained by a class of students in Statistics (out of 100): 15

Paper I	45	55	56	58	60	65	68	70	75	80	85
Paper II	56	50	48	60	62	64	65	70	74	82	90

Compute the co-efficient of correlation for the above data. Find also the equations of the lines of regression.

6. What do you mean by conditional probability? Let E be an event for which $P(E) \geq 0$. Show that the conditional probability function $P(*|E)$ satisfies the axioms of a probability space, that is: 10

(a) For any events A, we have $P(A|E) \geq 0$

(b) For any certain event S, we have $P(S \mid E)=1$

(c) For any two disjoint events A and B, we have,

$$P((A \cup B) \mid E) = P(A \mid E)+P(B|E).$$

(d) For any infinite sequence of mutually disjoint events $A_1, A_2, A_3, \ldots\ldots$, we have

$$P(A_1 \cup A_2 \cup A_3 \ldots\ldots|E) = P(A_1|E) + P(A_2|E) + P(A_3|E) + \ldots..$$

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