2018

MCA

MCA : 2.4

GRAPH THEORY

Full Marks: 75

Time: 3 Hour

The figures in the margin indicate full marks for the questions

1. Attempt the following parts: **1x5=5**

a. A graph G is called a .....if it is a connected acyclic graph .

i. Cyclic graph

ii. Regular graph

iii. Tree

iv. Not a graph.

b. A graph with n vertices will definitely have a parallel edge or self loop if the number of edges are,

i. Greater then (n-1).

ii. Less than n(n-1).

iii. Greater than $\frac{n(n-1)}{2}$.

iv. Less than $\frac{n^2}{2}$.

c. Graph is a collection of,

i. Rows and columns

ii. Vertices and edges

iii. Equations

iv. None of these.

d. A minimal spanning tree of a graph G is,

i. A spanning tree sub-graph.

ii. A tree

iii. Minimum weights.

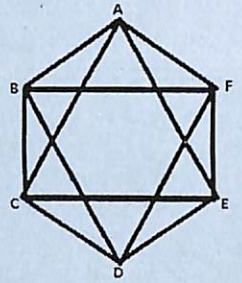
iv. All of the above.

e. An undirected graph possesses an Eulerian if and only if it is connected and its vertices are
   i. All of even degree
   ii. All of odd degree
   iii. All of any degree
   iv. Even in numbers.

2. Attempt the following parts: 2x10=20

a. Prove that in a graph, the number of vertices of odd degree is even.

b. Draw a planar representation of the given graph:



c. Suppose that in a graph of 5 peoples, like A,B,C,D and E, the following pairs of people are acquainted with each other, A & C, A & D, B & C, C & D, C & E. Drawn adjacency matrix for the graph and find the degree of the graph.

d. Suppose that G is a simple connected planar graph drawn n, so that no edges cross with $n \geq 3$ vertices and e edges and that the graph divides the plane into r regions. Then show that $e \leq 3n - 6$.

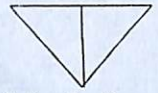
e. Give example of a graph having Euler's circuit but not Hamiltonian circuit.

f. Let $G = (V, E)$ be a connected undirected graph. What is the largest possible value for $|V|$ if $|E| = 19$ and $\deg(v) \geq 4, \forall\, v \in V$.

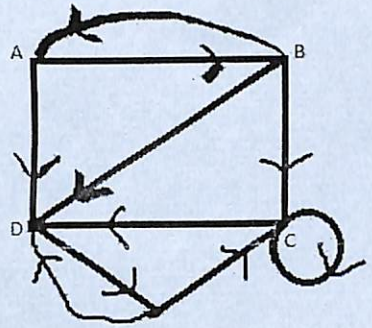
g. Define vertex connectivity and edge connectivity of a graph..

h. What do you mean by chromatic number for a coloring graph?

i. Find all spanning tree of the following graph:

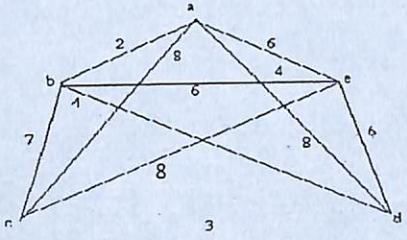


j. Find the in-degree and out-degree of the give graph. Are there any sink and source? Find all simple path from A to C.



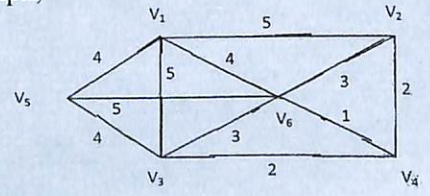
3. Attempt any five of the following parts: 4x5=20

a. Define eccentricity of the vertex and centre of a graph? Find the centre of the graph of the given vertices.



b. Prove that a simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges.

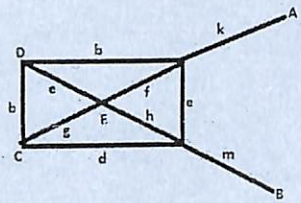
c. Using Kruskal's algorithm to find out the minimal spanning tree of the following graph,



d. Show that for every connected graph has at least one spanning tree.

e. Show that a graph with n vertices, (n-1) edges and no circuits is connected.

f. What do you mean by incidence matrix and circuit matrix? Find incidence and circuit matrix from the following graph.

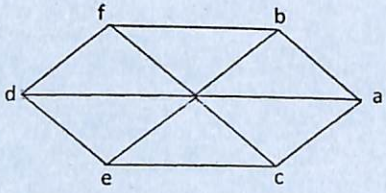


4. Attempt all parts of the following: **6x5=30**

a. What do you mean by Euler graph and Hamiltonian circuit. Show that a connected graph G is an Euler graph if and only if all vertices are of even degree.

b. State and prove Cayley's theorem for counting tree.

c. What do you mean by chromatic polynomial of a graph? Determine the chromatic polynomial of the following graph,



d. State and prove five color theorem for planner graph.

e. What do you mean by connected and regular graph in diagraph. Apply Dijkstra algorithm to find out the shortest path from the vertex a to every other vertices in the following graph.