2016

MCA

MCA 2.4

GRAPH THEORY

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Attempt the following parts : $1 \times 5 = 5$

   a. The minimum number of edges in a connected graph with n vertices is,

      i) (n-1)

      ii) n

      iii) (n+1)

      iv) None of these

   b. In a directed graph,

      i) Direction are fixed

      ii) Underlying is fixed

      iii) Both (i) and (ii)

      iv) None of (i) and (ii)

   c. Maximum number of edges in an n-nodes undirected complete graph is,

      i) n^2

      ii) $\frac{n(n-1)}{2}$

iii) $(n-1)$

iv) $\frac{n(n+1)}{2}$

d. Consider a simple connected graph with n vertices and n edges (n > 2). Then which of the following statements are true,
   i) G has a cycle.
   ii) G has at least one cycle.
   iii) The graph obtained by removing any edge from G is not connected.
   iv) None.

e. A connected acyclic graph G is
   i) Acyclic graph
   ii) Tree
   iii) Open graph
   iv) Close graph

2. Attempt any five parts of the following : $2 \times 5 = 10$

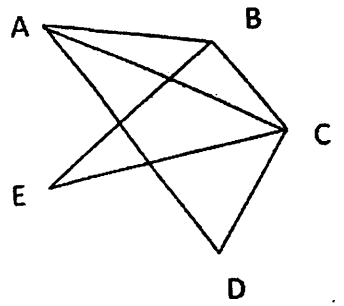
a. Define complement of a graph and infinite graph with example.

b. Show that there is only one path between every pair of vertices in a tree.

c. Give an example of a graph having Euler's circuit and Hamiltonian circuit both.

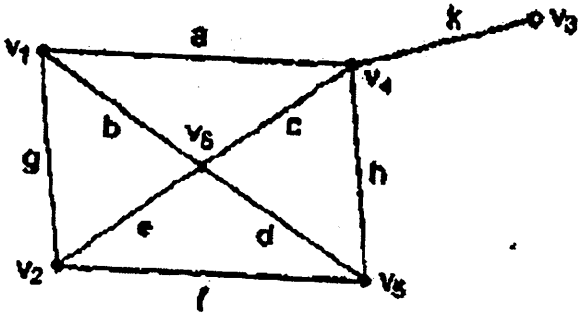
d. Define edge connectivity and vertex connectivity of a graph.

e. Consider the below graph G



Find the degree of each vertex and verify that the sum of the degrees of the vertices twice of the number of edges.

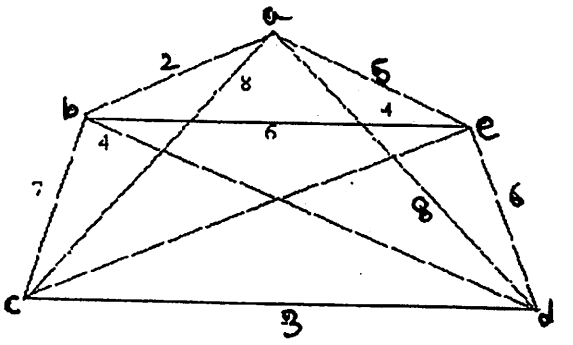
f. List all cut-sets with respect to the following graph.



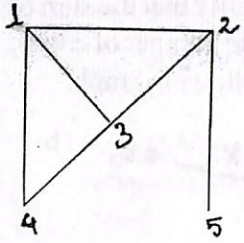
3. Attempt any Six parts of the following : $4 \times 6 = 24$

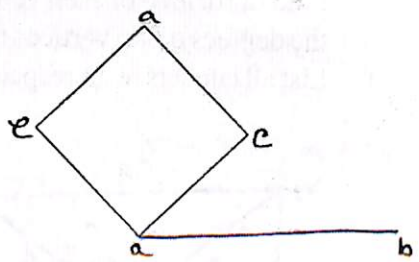
a. Show that tree with n vertices has (n-1) edges.

b. Apply Prime's algorithm to find a minimal spanning tree of the following graph.



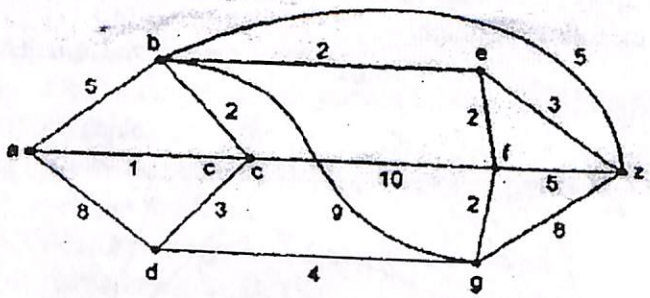
c. When two graphs are said to be isomorphic? Whether the following graphs are isomorphic or not. Explain your answer.





d. Prove that a simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges.

e. Apply Dijkstra algorithm to find out the shortest path from the vertices a to z in the following graph



f. Show that if a connected graph is an Euler graph then all vertices of G are of even degree.

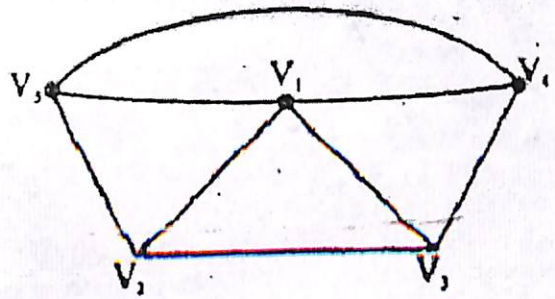
g. Define network flows. Prove that maximum flow possible between two vertices a and b in network is equal to the minimum of capacities of all cut-sets respect to a and b.

4. Attempt all parts of the following : $6 \times 6 = 36$

a. State and prove the five colour theorem of a graph.

b. Define label graph. Show that the number of n-vertices labeled trees is n^{n-2}, for $n \geq 2$.

c. Define the Chromatic polynomial of a graph G. Find the Chromatic polynomial of the following graph.



d. Give the definition of planar graph. Show that the complete graph of five vertices is non-planar.

e. Write down the steps of elementary reduction for detecting the planarity.

f. Define following with one example each.

i) Circuit matrix

ii) Path matrix

iii) Adjacency matrix.

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