2023

MATHEMATICS

Paper : MATSE3012

(Analytical Geometry)

Full Marks : 50

Pass Marks : 20

Time : 2 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer (any *five*) : 1×5=5

(a) The focus and vertex of a parabola $y^2 = 4x$ are

(i) (2, 0), (0, 2)

(ii) (1, 0), (0, 0)

(iii) (0, 1), (0, 0)

(iv) (0, 2), (0, 0)

(b) Under what condition a general equation of second degree

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

represents a hyperbola?

(i) $ab - h^2 = 0$

(ii) $ab - h^2 > 0$

(iii) $ab - h^2 < 0$

(iv) $a = b$ and $h = 0$

(c) What is the eccentricity of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$?

(i) $\frac{7}{4}$

(ii) $\frac{\sqrt{7}}{4}$

(iii) $\frac{7}{16}$

(iv) $\sqrt{\frac{7}{4}}$

(d) The radius of the sphere given by

$$x^2+y^2+z^2+3x-4y+5z+5=0$$

is

(i) $\frac{1}{2}\sqrt{30}$

(ii) $\sqrt{30}$

(iii) $\frac{\sqrt{30}}{4}$

(iv) $\frac{3}{2}$

(e) What is the nature of the following surface given by the equation :

$$9x^2+36y^2+4z^2-36x+216y+32z+388=0?$$

(i) A come

(ii) A paraboloid

(iii) An ellipsoid

(iv) A hyperboloid

(f) The eccentricity of the ellipse

$$\frac{x^2}{25}+\frac{y^2}{9}=1$$

is

(i) 25

(ii) $\frac{5}{4}$

(iii) $\frac{4}{5}$

(iv) 9

(g) The length of the latus rectum of the parabola $y^2 = 12x$ is

(i) 12

(ii) 4

(iii) 10

(iv) 6

(h) The focii and vertices of the hyperbola

$$\frac{x^2}{16}-\frac{y^2}{9}=1$$

are

(i) (±5, 0), (0, 0)

(ii) (±5, 0), (±4, 0)

(iii) (±2, 0), (±4, 0)

(iv) (±3, 0), (±4, 0)

(i) The equation of the ellipse where given vertices (±6, 0) and focii (±4, 0) is

(i) $\frac{x^2}{9}+\frac{y^2}{25}=1$

(ii) $\frac{x^2}{25}+\frac{y^2}{9}=1$

(iii) $\frac{x^2}{36}+\frac{y^2}{20}=1$

(iv) $\frac{x^2}{20}+\frac{y^2}{36}=1$

(j) The equation of the hyperbola where vertices (0, ± 5), focii (0, ± 8) is

(i) $\frac{y^2}{25}-\frac{x^2}{39}=1$ (ii) $\frac{y^2}{25}-\frac{x^2}{9}=1$

(iii) $\frac{y^2}{16}-\frac{x^2}{9}=1$ (iv) $\frac{y^2}{4}-\frac{x^2}{5}=1$

2. Answer any *five* of the following questions : 2×5=10

(a) If $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ be the extremities of any focal chord, prove that $t_1t_2=-1$.

(b) Find the condition that the line $lx+my=n$ is a tangent to the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

(c) Sketch the curve : $x=(y-2)^2+2$

(d) Find the eccentricity and the focii of the hyperbola $4x^2-9y^2=36$.

(e) Find the equation of the sphere whose centre is (1, 2, 3) and radius 4.

(f) Define a cone. What do you mean by the vertex and the guiding curve of a cone?

(g) Find the centre and radius of the sphere

$$x^2+y^2+z^2+2x-4y+2z-3=0$$

3. Answer any *five* of the following questions : 5×5=25

(a) Find the equation of the parabola whose focus is the origin and whose directrix is the line $2x+y-1=0$ and sketch the parabola.

(b) Prove that the line $lx+my=n$ is a normal to the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ if } \frac{a^2}{l^2}+\frac{b^2}{m^2}=\frac{(a^2-b^2)^2}{n^2}$$

(c) If the line $x\cos\alpha+y\sin\alpha=\rho$ touches the ellipse $\frac{y^2}{a^2}+\frac{x^2}{b^2}=1$, show that

$$a^2\cos\alpha+b^2\sin\alpha=p^2$$

(d) Reduce the following equation of a conic to its standard form :

$$14x^2+4xy+11y^2-44x-58y+71=0$$

(e) Find the equation of the sphere through the circle $x^2+y^2+z^2=25$, $x+2y-z+2=0$ and the point (1, 1, 1).

(f) Find the equation of the cone whose vertex is (α, β, γ) and the guiding curve is the conic $z=0$, $f(x, y)=0$

(g) The axis of a right circular cylinder is

$$\frac{x-1}{2}=\frac{y-2}{-1}=\frac{z-3}{2}$$

and its radius of 5. Find its equation.

(h) Find the equation of the cylinder generated by the lines parallel to the line

$$\frac{x}{1}=\frac{y}{2}=\frac{z}{1}$$

and intersecting the guiding curve $z=3$, $x^2+y^2=4$.

(i) Write down the conditions under which the equation

$$ax^2+2hxy+by^2+2gx+2fy+c=0$$

represents—

(i) a parabola;

(ii) an ellipse;

(iii) a hyperbola;

(iv) a circle;

(v) a pair of straight lines.

4. Answer any *one* of the following questions : 10

(a) *(i)* Prove that the product of perpendiculars from focii to any tangent to the hyperbola

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

is constant. 5

(ii) Prove that there is no portion of the ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

beyond the two planes $z=+c$ and $z=-c$ 5

(b) (i) Find the equation of the cylinder generated by lines parallel to the lines

$$\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$$

whose guiding curve is the conic

$$z=0,\ ax^2+2hxy+by^2=1$$ 5

(ii) Find the equation of the sphere passing through the origin and the points at which the plane

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

touches the coordinate axes. Hence find the centre and radius of the sphere. 5

(c) Prove that the plane $ax+by+cz=0$ cuts the cone $yz+zx+xy=0$ in perpendicular generators, if

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$$ 10

★★★