63/1 (SEM-3) CC7/BCAHC3076

2023

COMPUTER APPLICATION

Paper : BCAHC3076

(Mathematics—II)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer from the following (any *six*) : 1×6=6

(a) The number of ways of selecting one or more object(s) out of n distinct objects is

(i) 2^n

(ii) $2^n - 1$

(iii) $\underline{|n}$

(iv) n^2

(b) If A and B are two sets such that $n(A) = 2$ and $n(B) = 3$, then the number of relations from A to B is

(i) 64

(ii) 32

(iii) 6

(iv) 16

(c) The number of vertices of odd degree in an undirected graph is

(i) odd

(ii) any real number

(iii) even

(iv) a prime

(d) Let A and B be two sets such that $n(A) = m$ and $B = \phi$. The number of elements in $P(A \times B)$ is

(i) 0

(ii) 1

(iii) 2^m

(iv) m

(e) The function $f : \mathbb{R} \to \mathbb{R}$, defined by $f(x) = |x|$, is

(i) one-one

(ii) onto

(iii) one-one but not onto

(iv) neither one-one nor onto

(f) The relation $R = \{(2, 2), (3, 3), (1, 2), (2, 1)\}$ on the set $A = \{1, 2, 3\}$ is

(i) symmetric

(ii) transitive

(iii) reflexive

(iv) both symmetric and transitive

(g) Let A and B be two sets such that $n(A) = m$, $n(B) = n$ with $m > n$. The number of injective functions from A to B is

(i) 0

(ii) m

(iii) n

(iv) mn

(h) 8th term of the progression 4, 2, 1, $\frac{1}{2}$, ... is

(i) $\frac{1}{32}$

(ii) $\frac{1}{16}$

(iii) $\frac{1}{64}$

(iv) $\frac{1}{124}$

(i) The value of $^5C_3$ + $^5C_4$ is

(i) 5

(ii) 10

(iii) 15

(iv) 20

(j) The maximum number of edges of a simple graph having n vertices is

(i) n

(ii) $n-1$

(iii) $\frac{n(n-1)}{2}$

(iv) $n(n-1)$

2. Answer any *five* of the following questions : 2×5=10

(a) Define $f, g, h : \mathbb{R} \to \mathbb{R}$ by $f(x) = x^4$, $g(x) = x^3 - 4x$ and $h(x) = \frac{1}{x^2+1}$. Find $f \circ h(x)$ and $g \circ h(x)$.

(b) Let A, B and C be three sets. If $A \in B$ and $B \subset C$, is it true $A \subset C$? If not, give a counterexample.

(c) Define null set. Show that it is a subset of every set.

(d) Find the number of 5-digit telephone number having at least one of their digits repeated.

(e) Define complete graph. What is the maximum number of edges of a complete graph?

(f) Is 5, 5, 4, 3, 2, 1 the degree sequence for a graph? If not, explain why no graph exists.

(g) Show that the proposition $p \vee (q \to p)$ and $p \vee (p \to q)$ is tautology.

3. Answer any *six* of the following questions : 5×6=30

(a) If $n \geq 0$, prove that

$$\sum_{i=0}^{n} ({}^nC_i)^2 = {}^{2n}C_n$$

(b) If a, b, c are in AP, then prove that

$$a^3 + 4b^3 + c^3 = 3b(a^2 + c^2)$$

(c) Find a formula for the general term F_n of the Fibonacci sequence 0, 1, 1, 2, 3, 5, 8, 13, ...

(d) Find the CNF of the statement

$$(p \wedge \sim (q \wedge r)) \vee (p \rightarrow q)$$

(e) Let A and B be two sets. If $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for some set X, show that $A = B$.

(f) For any two positive numbers, show that $AM \geq GM$.

(g) Let R be a relation on the set Z defined by $R = \{(x, y) \mid x - y$ is divisible by 6, $x, y \in z\}$. Show that R is an equivalence relation.

(h) Find the sum of the series 4 + 44 + 444 + ... to n-terms.

(i) Construct a truth table for the compound proposition

$$(\sim p \leftrightarrow \sim q) \leftrightarrow (p \leftrightarrow q)$$

and state whether it is tautology or contradiction or contingency.

(j) Show that the limit of a convergent sequence is unique.

4. Answer any *two* of the following questions : 10×2=20

(a) The $(m+n)$th term and $(m-n)$th term of a GP are p and q respectively. Show that mth and nth term of the GP are $\sqrt{pq}$ and $p\left(\frac{q}{p}\right)^{\frac{m}{2n}}$ respectively.

(b) Write the breadth-first search algorithm.

(c) Prove that the necessary and sufficient condition for the function $f : A \rightarrow B$ to be invertible is that f is bijective.

(d) Test convergence of the following :

(i) $\sum u_n$, where $u_n = \sqrt{n+1} - \sqrt{n}$

(ii) $x + \frac{3}{5}x^2 + \frac{8}{10}x^3 + \ldots$

5. Answer any *one* of the following questions : 14

(a) Define bipartite graph. Draw the graph $K_{3,5}$. Prove that the number of edges in a bipartite graph with n vertices is at most $\frac{n^2}{2}$. 3+3+8=14

(b) (i) $(A \cup B)' = A' \cap B'$ where A and B are any two sets. 4

(ii) Prove that, $A - B = A \cap B' = B' - A'$ where A and B are any sets. 5

(iii) 75% students of a college play cricket and 40% play football. Find the percentage of students who play both football and cricket. 5

(c) What is exponential and logarithmic series? Explain logarithmic series expansion. (4+4)+6=14

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