63/1 (SEM–3) CC5/PHYHC3056

2023

PHYSICS

Paper : PHYHC3056

(Mathematical Physics—II)

Full Marks : 60

Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct option from the following (any *five*) : 1×5=5

(a) A function $f(x)$ is said to be even or symmetric if

(i) $f(-x) = f(x)$

(ii) $f(-x) = -f(x)$

(iii) $f(x) = -f(x)$

(iv) $f(x) = 0$

(b) $\int_0^{2\pi} \cos^2 nx\, dx =$

(i) π

(ii) $-\pi$

(iii) 0

(iv) 1

(c) The regular singular point of the equation $y'' + x^2 y = 0$ is

(i) 0

(ii) 1

(iii) -1

(iv) 2

(d) $\Gamma(n+1) =$

(i) $(n+1)!$

(ii) $n!$

(iii) $(n-1)!$

(iv) $\frac{n!}{2}$

(e) $\beta(l, m) =$

(i) $\frac{\Gamma(l)\cdot\Gamma(m)}{\Gamma(l+m)}$

(ii) $\frac{\Gamma(l+m)}{\Gamma(l)}$

(iii) $\Gamma(l)\Gamma(m)$

(iv) $\frac{\Gamma(l)\Gamma(m)}{\Gamma(l-m)}$

(f) Standard Error (SE) =

(i) $\frac{\sqrt{n}}{\text{SD}}$

(ii) $\frac{\sqrt{n-1}}{\text{SD}}$

(iii) $\frac{\text{SD}}{\sqrt{n}}$

(iv) $\frac{(\sqrt{n+1})}{\text{SD}}$

(g) Probable error =

(i) $\pi \times$ SD

(ii) 0·6745 × SD

(iii) 0·6745 × SE

(iv) 0·6745 × π

(h) $H_0(x) =$

(i) 0

(ii) 1

(iii) −1

(iv) x

(i) If $x = (1 \cdot 0 \pm 0 \cdot 1)$ cm, $y = (2 \cdot 0 \pm 0 \cdot 2)$ cm and $z = (4 \cdot 0 \pm 0 \cdot 01)$ cm, then the uncertainty in q, where $q = x + y - z$ is

(i) 0·050

(ii) 0·060

(iii) 0·005

(iv) 0·0035

(j) If $\delta x_1 = 1$ mm and $\delta x_2 = 2$ mm are the errors of x_1 and x_2 respectively, then the error in the sum of x_1 and x_2 is

(i) 2

(ii) 2·23

(iii) $\sqrt{5}$

(iv) $\sqrt{6}$

2. Answer any *five* of the following questions : 2×5=10

(a) Find the value of a_0 for odd function of Fourier series.

(b) Prove whether Frobenius method is applicable or not for the following second-order differential equation :

$$2x^2 \frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + (x^2 - 4)y = 0$$

(c) Find $\Gamma\left(-\frac{1}{2}\right)$, $\Gamma\left(\frac{5}{2}\right)$.

(d) Find $\int_0^\infty \sqrt{x}\, e^{-x}\, dx$.

(e) Prove that $\Gamma(n) = (n-1)!$.

(f) Find $\beta(m+1, n)+\beta(m, n+1)$.

(g) How can the stochastic errors be minimized?

3. Answer any *five* of the following questions : 5×5=25

(a) Find the value of a_0 and b_n for Fourier series

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cosh x+\sum_{n=2}^{\infty} b_n \sinh x$$

(b) Find the recurrance relation for the differential equation

$$\frac{d^2y}{dx^2}+y=0$$

(c) Find $P_n(1)$ from the following function of Legendre polynomial

$$(1-2xz+z^2)^{-\frac{1}{2}}=\sum_{n=0}^{\infty} P_n(x) z^n$$

(d) Find $H_2(x)$, $H_3(x)$ from

$$H_n(x)=\sum_{r=0}^{N}(-1)^r \frac{n!(2x)^{n-2r}}{r!(n-2r)!}$$

where, $N=\frac{n}{2}$, if n is even

$=\left(\frac{n-1}{2}\right)$, if n is odd 2½×2=5

(e) Show that

$$\frac{\Gamma\left(n+\frac{1}{2}\right)2^n}{\sqrt{\pi}}=1.3.5\cdots(2n-1)$$

(f) If $\sum H_n(x)\frac{t^n}{n!}=e^{2tx-t^2}$, then show that

(i) $H_n'(x)=2n H_{n-1}(x)$

(ii) $2xH_n(x)=2nH_{n-1}(x)+H_{n+1}(x)$ 2½×2=5

(g) The height of five poles are 22·8 mm, 23·1 mm, 22·7 mm, 22·6 mm, 23·0 mm. Find the standard error.

(h) In the experiment of simple pendulum, the length and time period of the pendulum are $l=92\cdot95\pm0\cdot2$ cm and $T=1\cdot936\pm0\cdot003$ sec. Find the error in the measurement of g.

(i) Solve the problem using the method of separation of variables

$$\frac{\partial u}{\partial x}=2\frac{\partial u}{\partial t}+u$$

where, $u(x, 0)=6e^{-3x}$.

4. Answer any *two* of the following questions : 10×2=20

(a) If $f(x) = (x + x^2)$ for $-\pi < x < \pi$, find Fourier expression and show that

$$\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$$ 7+3=10

(b) Find the power series solution of the following differential equation :

$$y'' + 2xy' + 2\lambda y = 0$$

(c) Show that

$$P_n(x) = \frac{1}{2^n \cdot n!} \cdot \frac{d^n}{dx^n}(x^2 - 1)^n$$

from the Legendre differential equation and also find $P_1(x)$ and $P_2(x)$. 8+2=10

(d) Find the solution of the wave equation

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

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