2023

STATISTICS

Paper : STSHC3076

(Mathematical Analysis)

Full Marks : 60
Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer from the following (any *five*) : 1×5=5

(a) The set $\{x : a \leq x \leq b\}$ consisting of a, b and all real numbers lying between a and b is called

(i) semi-closed interval

(ii) semi-open interval

(iii) closed interval

(iv) open interval

(b) If $S_n = \left\{\frac{(-1)^n}{n}, n \in N\right\}$, then

(i) -1 is the infimum and $\frac{1}{2}$ is the supremum

(ii) $\frac{1}{2}$ is the infimum and -1 is the supremum

(iii) 0 is the infimum and -1 is the supremum

(iv) -1 is the infimum and 0 is the supremum

(c) The $\lim_{n\to\infty} \frac{1+2+3+\cdots+n}{n^2}$ equals

(i) 0

(ii) $\frac{1}{2}$

(iii) 2

(iv) 1

(d) The function $f(x) = x^2 + 3, \quad x \in [-2, 2]$, the value of c for Rolle's theorem is

(i) 1

(ii) 2

(iii) 0

(iv) -1

(Continued)

(e) If Σu_n is a positive term series, such that

$$\lim_{n\to\infty} n\left(\frac{u_n}{u_n+1} - 1\right) = l$$

then the series

(i) converges if $l > 1$ and diverges if $l < 1$

(ii) diverges if $l > 1$ and converges if $l < 1$

(iii) converges if $l \geq 1$ and diverges if $l \leq 1$

(iv) diverges if $l \geq 1$ and converges if $l \leq 1$

(f) If x^2 is any constant, then $\Delta^2(x^2)$ is

(i) 2

(ii) $2x$

(iii) x

(iv) 0

(Turn Over)

(g) If $f(x)=\frac{x-4}{2\sqrt{x}}$, then $f'(1)$ is

(i) $\frac{5}{4}$

(ii) $\frac{4}{5}$

(iii) 1

(iv) 0

(h) The value of $\Delta^2(x+1)$, the interval of differencing being unity, is

(i) $2x$

(ii) 0

(iii) 1

(iv) 4

(i) The nth difference of a polynomial of degree n is constant, then $(n+1)$th difference is

(i) n

(ii) $n!$

(iii) zero

(iv) one

(j) Simpson's one-third rule, the integrand is assumed to be a polynomial of

(i) 1st degree

(ii) 2nd degree

(iii) 3rd degree

(iv) 4th degree

2. Answer any *five* of the following questions : 2×5=10

(a) Define derived set.

(b) Show that the set $s=\{x: 0<x<1, x\in R\}$ is open but not closed.

(c) Establish the relation between Δ and E.

(d) State Rolle's theorem.

(e) Prove that $E^2X^2 \neq (EX)^2$.

(f) Show that the series $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\cdots$ is not convergent.

(g) State D'Alembert's ratio test.

(b) State and prove Simpson's $\frac{1}{3}$rd rule. Using Simpson's $\frac{1}{3}$rd rule, find the value of

$$\int_0^6 \frac{1}{1+x} dx$$

(c) Prove that a necessary and sufficient condition for the convergence of a sequence $\{S_n\}$ is that for each $\varepsilon > 0$ there exists a positive integer m such that $|S_{n+p} - S_n| < \varepsilon \; \forall \; n \geq m$ and $p \geq 1$.

(d) State and prove Lagrange's interpolation formula. Apply this formula to evaluate the value of $f(4)$ from the following data :

x	:	1	3	7	12
$f(x)$	:	4	18	20	25

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