63/1 (SEM-3) CC5/STSHC3056

2023

STATISTICS

Paper : STSHC3056

(Sampling Distributions)

Full Marks : 60

Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer any *five* from the following questions : 1×5=5

(a) If we draw a sample of size n from a given finite population of size N, then the total number of possible samples is

(i) $^{N}C_{n} = \dfrac{N!}{(N-n)!}$

(ii) $^{N}C_{n} = \dfrac{N!}{n!\,(N-n)!}$

(iii) $^{N}C_{n} = \dfrac{n!}{(N-n)!\,N!}$

(iv) $^{N}C_{n} = \dfrac{N!}{(n-1)!\,(N-1)!}$

(b) If $X_1, X_2, X_3, \ldots, X_n$ is a sequence of random variables and if mean μ_n and standard deviation σ_n of X_n exists for all n and if $\sigma_n \to 0$ as $n \to \infty$, then

(i) $X_n - \mu \xrightarrow{p} 0$

(ii) $X_n - \mu \xrightarrow{p} 1$

(iii) $X_n - \mu \xrightarrow{p} -1$

(iv) All of the above

(c) Area of the critical region depends on

(i) size of type-I error

(ii) size of type-II error

(iii) value of statistics

(iv) number of observations

(d) A confidence interval of confidence coefficient $(1-\alpha)$ is best which has

(i) smallest width

(ii) vastest width

(iii) upper and lower limits equidistant from the parameter

(iv) one-sided confidence interval

(e) The ratio of between sample variance and within sample variance follows

(i) F-distribution

(ii) χ^2-distribution

(iii) z-distribution

(iv) t-distribution

(f) Mode of chi-square (χ^2) distribution is

(i) n

(ii) $2n$

(iii) $n-2$

(iv) n^2

(g) For two population $N(\mu_1, \sigma^2)$ and $N(\mu_2, \sigma^2)$ with σ^2 unknown, the test statistic for testing $H_0 : \mu_1 = \mu_2$ based on small samples with usual notations is

(i) $t = \dfrac{\bar{X}_1 - \bar{X}_2}{\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$

(ii) $t = \dfrac{\bar{X}_1 - \bar{X}_2}{\sqrt{\sigma^2\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$

(iii) $t = \dfrac{\bar{X}_1 - \bar{X}_2}{S_p\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$

(iv) Any of the above

(h) Which test is used for testing the difference between two means in a small sample?

(i) z-test

(ii) t-test

(iii) χ^2-test

(iv) F-test

(i) Degrees of freedom is related to

(i) number of observations

(ii) hypothesis under test

(iii) number of independent observations in a test

(iv) None of the above

(j) The cumulative distribution $F_1(x)$ of the smallest order statistic $X_{(1)}$ is given by

(i) $[F(x)]^n$

(ii) $[1-F(x)]^n$

(iii) $1-[1-F(x)]^n$

(iv) $1-[F(x)-1]^n$

(Continued)

2. Answer any *five* questions from the following : 2×5=10

(a) Define order statistics.

(b) Distinguish between parameter and statistic.

(c) Define type–I and type–II error.

(d) Write down the definition of Chebyshev's inequality.

(e) Mention any two applications of χ^2-distribution.

(f) What are the properties of Student's t-test?

(g) Define F-distribution.

3. Answer any *five* questions from the following : 5×5=25

(a) In $F(n_1, n_2)$ distribution, if $n_2 \to \infty$, then prove that $\chi^2 = n_1 F$ follows χ^2-distribution with n_1 d.f.

(b) State and prove Chebyshev's inequality for continuous variables.

(c) Write down the procedure for testing of hypothesis.

(d) Define (i) critical region, (ii) level of significance, (iii) degrees of freedom.

(Turn Over)

(e) Derive the distribution of rth order statistic.

(f) For χ^2-distribution, prove that in usual notations

$$M_{\chi^2}(t) = (1-2t)^{-\frac{n}{2}}$$

(g) Prove that as $n \to \infty$ the p.d.f. of t-distribution with n d.f. is

$$\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{t^2}{2}\right); -\infty < t < \infty$$

(h) Give the statement of Liapunov theorem.

(i) Mention all applications of t-distribution.

4. Answer any *two* questions from the following : 10×2=20

(a) State and prove W.L.L.N. (weak law of large numbers).

(b) Give cumulative distribution function of all (largest, smallest) single-order statistics.

(c) Derive χ^2-variate.

(d) If X is a chi-square variate with n d.f. then prove that for large n, $\sqrt{2X} \sim N(\sqrt{2N}, 1)$.

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