2015

PHYSICS

Paper : 101 (Old Course)

MATHEMATICAL PHYSICS

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1 **Answer the following** (any six) $1 \times 6 = 6$

(a) Eigen value of the matrix $A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

(i) $e^{\pm i\theta}$ (ii) $e^{\pm 2i\theta}$ (iii) $e^{\pm 3i\theta}$ (iv) $e^{\pm i\frac{\theta}{2}}$

(b) The eigenvalues of a Hermitian matrix are -
(i) Always zero (ii) Complex
(iii) Real (iv) None of these

(c) The value of Cristoffel's symbol of 2nd kind. $\left\{ \begin{matrix} s \\ pq \end{matrix} \right\}$

(i) $g^{sr}[pq, r]$ (ii) $g^{sr}[qp, r]$ (iii) $g^{sr}[rp, q]$ (iii) $g^{sr}[pr, q]$

(d) If A is a real symmetric matrix, the eigenvalues of A are
(i) Complex (ii) Real (iii) of unit magnitude (iv) None of these

(e) Legendre polynomials $P_1(x)$ is
(i) 1 (ii) x (iii) $\frac{1}{2}(3x^2 - 1)$ (iv) None of these

(f) The complex Fourier transformation of a function $f(x)$ is defined as

(i) $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{isx}\mathrm{d}x$ (ii) $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{-isx}\mathrm{d}x$
(iii) $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(-x)e^{-isx}\mathrm{d}x$ (iv) None of these

(g) The trace of a 3×3 matrix is 1. Two of its eigen values are 1 and 1. The third eigen value is -
(i) -1 (ii) 0 (iii) 1 (iv) 2

2 **Answer the following** (any five) $2 \times 5 = 10$

(a) Evaluate $\delta^p_q A^{qr}_s$ and $\delta^p_q \delta^q_r$.

(b) Calculate the residue of $f(z) = \frac{1}{z^2(z^2+1)}$ at $z = i$

(c) If X = $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and Y = $\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$, $X, Y \in R^3$, calculate $||x||$ and $\langle X, Y \rangle$ The symbols have their usual meanings.

(d) If $F(s)$ is the complex Fourier transform of $f(x)$, then prove that

$$F\{f(ax)\} = \frac{1}{a} F\left(\frac{s}{a}\right)$$

(e) Show that an $n \times n$ orthogonal matrix has $\frac{n(n-1)}{2}$ independent parameters.

(f) How many generators will SU(3) have ? Give an example in physics where SU(3) symmetry is observed.

(g) Prove that the eigen values of anti-Hermitian operator are imaginary.

(h) A quantity $A(j, k, l, m)$ which is a function of coordinates x^i transforms to another coordinate system x'^i according to the rule

$$A'(p, q, r, s) = \frac{\partial x^j}{\partial x'^p} \frac{\partial x'^q}{\partial x^k} \frac{\partial x'^r}{\partial x^l} \frac{\partial x'^s}{\partial x^m} A(j, k, l, m)$$

(i) Is the quantity a tensor ? (ii) If so, write the tensor in suitable notation and give the contravariant and covariant order and rank.

3 **Answer the following** (any five) $5 \times 5 = 25$

(a) From series representation of Bessel function, show that

$$J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$$

(b) Write the law of transformation for the tensors (i) A^i_{jk}; (ii) B^{mn}_{ijk}; (iii) C^m. If A^{pq}_r and B^{pq}_r are tensors, prove that their sum and differences are tensors.

(c) Using tensorial expression find

$$\text{div}\, \vec{A}$$

in spherical polar coordinates.

(d) Discuss the properties of linear vector space in n dimensions.

(e) Show that $\int_C \frac{1+z}{z(2-z)} dz = \pi i$,

where C is a unit circle with center at the origin of the complex plan.

(f) Prove that

(i) $L(e^{at} \cosh bt) = \frac{s-a}{(s-a)^2 - b^2}$

(ii) $L[f'(t)] = sL[f(t)] - f(0)$, where $L[f(t)] = F(s)$

4 **Answer the following** (any three) $3 \times 9 = 27$

(a) Prove that $\int_{-1}^{1} P_n(x) P_m(x) dx = \frac{2}{2n+1} \delta_{nm}$

(b) If A_p is a tensor show that the covariant derivative of A_p with respect to x^q i.e. $A_{p,q} = \frac{\partial A_p}{\partial x^q} - \left\{ \begin{matrix} s \\ pq \end{matrix} \right\} A_s$ is a 2nd rank covariant tensor.

(c) Diagonalize the following matrix -

$$\begin{pmatrix} 1 & 2i \\ -2i & 1 \end{pmatrix}$$

(d) Show that $L\left[\frac{1}{t} f(t)\right] = \int_s^\infty F(s) ds$, where $L[f(t)] = F(s)$

Using Laplace transforms, solve the simultaneous differential equations

$$(D+1)y_1 + (D-1)y_2 = e^{-t}$$
$$(D+2)y_1 + (D+1)y_2 = e^t$$

where $D = \frac{d}{dt}$ and $y_1(0) = 1$ and $y_2(0) = 0$.

5 **Answer the following** (any one) $1 \times 12 = 12$

(a) Solve the wave equation

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$$

under the condition: $u = 0$ when $x = 0$ and $x = \pi$
$\frac{\partial u}{\partial t} = 0$ when $t = 0$ and $u(x, 0) = x$, $0 < x < \pi$

(b) Evaluate the following integrals using the calculus of residues

(i) $\int_0^{2\pi} \frac{\sin^2 \theta}{5 - 4\cos\theta} d\theta$

(ii) $\int_0^\infty \frac{dx}{(1+x^2)^3}$

(c) Show that permutation of three objects form a non-abelian group S_3 of order six. Obtain the matrix representation of the generators of SU(3).

—— × ——