2018

PHYSICS

PHY: 101

MATHEMATICAL PHYSICS – I

Full Marks: 80

Time: 3 hours

1. **Choose the correct answer from the following,** **1x5=5**

(i) The trace of a 3×3 matrix is 2. Two of its eigen values are 1 and 2. The third eigen value is,

(a) -1 (b) 0 (c) 1 (d) 2

(ii) Eigen values of the matrix

$$\begin{bmatrix} -3 & \sqrt{\frac{19}{4}}e^{i\pi/3} \\ \sqrt{\frac{19}{4}}e^{-i\pi/3} & 6 \end{bmatrix}$$ are,

(a) 7/2, 5/2 (b) -5/2, 3/2 (c) -7/2, $^{13}/_2$ (d) -5/2, -7/2

(iii) If P_0, P_1 and P_2 are the Legendre polynomials, then find the correct statement

(a) $P_2(x) = 3x\,P_1(x) + \frac{1}{2}P_o(x)$

(b) $P_2(x) = \frac{3}{2}x\,P_1(x) - \frac{1}{2}P_o(x)$

(c) $P_2(x) = 3x\,P_1(x) - \frac{1}{2}P_o(x)$

(d) $P_2(x) = \frac{1}{2}x\,P_1(x) + \frac{3}{2}P_o(x)$

(iv) The residue of the function $\frac{1}{(Z+a)}\frac{1}{(Z^2+b^2)}$ at $z = ib$ is,

(a) $\frac{1}{2b(a+ib)}$ (b) $\frac{-i}{2b(a+ib)}$ (c) $\frac{1}{2b(a+ib)}$

(d) $\frac{-1}{2b(a+ib)}$

(v) Bessel's function describes the

(a) Velocity of light (b) Modes of acoustic waves

(c) Momenta (d) None of the above

2 Answer the following questions (*any five*) 2x5=10

a) Find the imaginary part of the analytic function whose real part is $x^3 - 3xy^2 + 3x^2 - 3y^2$.

b) Show that the set $\{(1,1,0),(1,0,1),(0,1,1)\}$ is a basis of the vector space R^3.

c) Find the Laurent series of the function

$$f(z) = \frac{z}{(z+1)(z+2)} \text{ about } z = -2$$

d) Show that $J_{-n}(x) = (-1)^n J_n(x)$

e) Show that $\frac{d}{dx}(x^n J_n) = x^n J_{n-1}$

f) If $P_n(x)$ is the Legendre polynomial of degree n then prove that $P_n(1) = 1$.

g) Examine if the following operator A is linear:

$$Af(x) = (1-x^2)\frac{d^2f}{dx^2} - 5x\frac{df}{dx} + 7f(x).$$

3. Answer the following questions (*any five*) 3x5=15

a) Prove that $\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta \int_0^{\pi/2} \frac{1}{\sqrt{\sin\theta}} d\theta = \pi$

b) Find the complex number Z if $\arg(Z+1) = \frac{\pi}{6}$ and $\arg(Z-1) = \frac{2\pi}{3}$

c) Find the residue of $\frac{1}{(Z^2+1)^3}$ at $Z = i$.

d) Determine the value of α, β and γ when $\begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}$ is orthogonal.

e) Expand the function $f(z) = \frac{1}{z+1}$ in Taylor's series about $z = 1$.

f) Use Cauchy's integral formula to evaluate $\oint_C \frac{4-3z}{z(z-1)(z-2)} dz$, where C is a circle $|z| = \frac{3}{2}$.

4. Answer the following questions (any four) 5x4=20

a) Prove that the following recurrence relations

(i) $xJ_n'(x) = nJ_n(x) - xJ_{n+1}(x)$

(ii) $xJ_n(x) = x\,J_{n-1}(x) - nJ_n(x)$

b) Using generating function of Legendre polynomial, show that

$$nP_n = (2n-1)x\,P_{n-1} - (n-1)P_{n-2}$$

c) Solve $x^4 + i = 0$ and locate the indicated roots graphically.

d) Prove that $\int_0^1 x^m (\log x)^n dx = \frac{(-1)^n}{(m+1)^{n+1}}\Gamma(n+1)$

e) Prove that $\int_0^1 (1-x^n)^{1/2} = \frac{1}{n}\frac{\left[\Gamma\left(\frac{1}{2}\right)\right]^2}{2\Gamma\left(\frac{2}{n}\right)}$

f) Prove that $B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$.

5. **Answer the following questions (any three)** **3x10=30**

(a) Use complex variable technique to find the value of the integral

(i) $\int_0^{2\pi} \frac{\sin^2\theta}{5-4\cos\theta}\, d\theta$ 5+5=10

(ii) $\int_{-\infty}^{+\infty} \frac{x^2}{(x^2+1)(x^2+4)}\, dx$

(b) Prove the generating function of Hermite polynomial, and show that Hermite differential equation, 10

$$H_n(x) = (-1)^n\, e^{x^2} \frac{d^n}{dx^n} e^{-x^2}$$

(c) If α and β are the roots of $J_n(ax) = 0$, then show that.

$$\int_0^a x\, J_n(\alpha x) J_{n'}(\beta x) dx = 0$$ 10

(d) Define Fuch's theorem. Using power series method, find the solution of the following differential equation about $x = 0$, 2+8

$$y'' + xy' + (x^2 + 2)y = 0$$

(e) Determine whether the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ can be diagonalized? If yes, find the diagonal matrix similar to A and the matrix which diagonalizes A. 10

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