2018

PHYSICS

PHY: 102

CLASSICAL MECHANICS

Full Marks: 80

Time: 3 hours

The figures in the margin indicates full marks for the questions

1. Answer the following questions **5x1=5**

(a) Constraint in a rigid body is

(i) holonomic (ii) nonholonomic

(iii) rheonomic (iv) none of these

(b) If the generalized coordinate is angle θ, the corresponding generalized force has the dimension of

(i) displacement (ii) torque

(iii) momentum (iv) energy

(c) If the Hamiltonian of a system is $H = \frac{p^2}{2m} + V(q)$, the value of $[p, [p, H]]$ is

(i) 0 (ii) 1 (iii) $\frac{\partial^2 V}{\partial q^2}$ (iv) $\frac{\partial V}{\partial q}$

(d) Action-angle variable has the dimension of

(i) Wavelength (ii) angular momentum (iii) angular frequency (iv) time

(e) If the generating function F is a function of (q_k, Q_k, t), then

(i) $p_k = \frac{\partial F}{\partial q_k}, Q_k = \frac{\partial F}{\partial P_k}$ (ii) $p_k = -\frac{\partial F}{\partial q_k}, Q_k = \frac{\partial F}{\partial P_k}$

(iii) $p_k = \frac{\partial F}{\partial q_k}, P_k = -\frac{\partial F}{\partial Q_k}$ (iv) $p_k = -\frac{\partial F}{\partial q_k}, P_k = -\frac{\partial F}{\partial q_k}$

2. Answer the following questions 5x2=10

(a) Deduce Hamilton's equation of motion for linear harmonic oscillator. 2

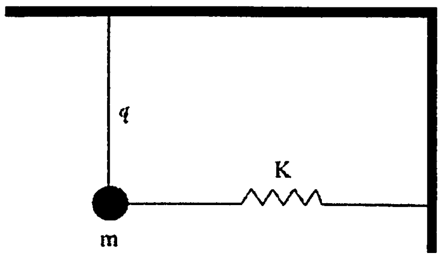
(b) For the time dependent canonical transformations $Q = p\tan t, P = -q\cot t$, calculate the generating function $F_1(q, Q, t)$ and $H'(Q, P)$. 2

(c) If $F_k = (q_1, q_2, q_3 \dots\dots q_n;\ p_1, p_2, p_3 \dots\dots p_n)$ Show that

$$\sum_{k=1}^{2n} \{F_k, F_i\}\,[F_k, F_j] = \delta_{ij}$$

Where $\{F_k, F_i\}$ is the Lagrange bracket and $[F_k, F_j]$ is Poisson bracket. 2

(d) A simple pendulum of mass m and length l is connected by a light spring of force constant k and the other end of the spring is fixed at the wall as shown in figure below. If the pendulum undergo small oscillation, find an expression for frequency of oscillation. 2



(e) Define linear and nonlinear system with one example to each. 2

3. Answer the following questions (any five) 5x5=25

(a) What are generalized coordinate and generalized velocities? Set up the Lagrange's equation of motion of a particle moving on the surface of the earth using spherical polar coordinates. 1+4

(b) Calculate the frequency of one dimensional linear harmonic oscillator by using the method of action-angle variable. Discuss the physical significance of Hamilton's characteristic function. 4+1

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(c) If the transformation equations $Q = q^m \cos np, P = q^m \sin np$ is a canonical transformation then using the property of fundamental Poisson bracket find the value of m and n. 5

(d) A simple pendulum of mass m is suspended with massless and inextensible string of length l. Obtain an expression for phase trajectory and illustrate graphically. 3+2

(e) For the bifurcation expressed by $\dot{x} = r + x^2$, sketch all vector fields that occur as r is varied. Find the bifurcation point and sketch the bifurcation diagram. $1\frac{1}{2} + 1\frac{1}{2} + 2$

(f) Derive Bernoulli's equation of fluid motion. What does Bernoulli's equation signifies? 4+1

4. Answer the following questions (any four) 4x10=40

(a) (i) Define Eulerian angles. Obtain the matrix of transformation from space coordinates to body coordinates in terms of Eulerian angles. 1+5

(ii) If I is the moment of intertia about the axis of rotation, prove that the kinetic energy can be expressed as 4

$$T = \frac{1}{2} I\omega^2$$

(b) Use Hamilton-Jacobi method to solve the Kepler's problem for a particle in an inverse square central force field. 10

(c) What is canonical transformation? Discuss the harmonic oscillator as an example of canonical transformations and explain graphically. 2+8

(d) Two pendula of equal mass m and length l are coupled with a spring of force constant k. If the coupled pendula undergo small oscillation, find the normal frequencies and normal coordinates of the oscillation. 5+5

(e) A double pendulum consists of a pendulum of mass m_1 and length l_1 to which the second pendulum of mass m_2 and length l_2 is suspended. If they undergo small oscillation in vertical plane, find the normal frequency of oscillation. What will be the frequencies if $m_1 = m_2$ and $l_1 = l_2$? 8+2

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