BU/EX/PG/SEM-1/103

2018

PHYSICS

PHY 103

QUANTUM MECHANICS - I

Full Marks- 80

Time – 3 hours

(The figures in the margin indicate full marks for the questions)

1. Answer the following: **$3 \times 5 = 15$**

(a) Define projection operator. Establish the condition under which the quantity $|\psi\rangle\langle\psi|$ will behave as projection operator.

(b) Show that the uncertainty product $\Delta x \Delta p$ is minimum $(= \hbar/2)$ for a Gaussian wave packet.

(c) Show that the sum of two projection operators cannot be a projection operator unless their product is zero.

(d) Consider a state $|\psi\rangle = \frac{1}{\sqrt{2}}|\phi_1\rangle + \frac{1}{\sqrt{5}}|\phi_2\rangle + \frac{1}{\sqrt{10}}|\phi_3\rangle$ which is given in terms of three orthonormal Eigenstates $|\phi_1\rangle, |\phi_2\rangle$ and $|\phi_3\rangle$ of an operator $\hat{B}$ such that $\hat{B}|\phi_n\rangle = n^2|\phi_n\rangle$. Find the expectation value of $\hat{B}$ for the state $|\psi\rangle$.

(e) Write down the properties of a well behaved wave function.

2. Answer the following: (any five) **$5 \times 5 = 25$**

(a) What do you mean by *collapse* of a wave function? The wave function of a free particle is represented by,

$$\psi(x) = A \exp\left(-\frac{x^2}{2a^2} + ikx\right).$$

Normalize the wave function.

P.T.O.

(b) Find the probability that a particle trapped in a box of width L can be found between 0.45L and 0.55L for the ground and first excited state.

(c) Electron with energy 1 eV incident on a barrier 5 eV and width 5Å. Find the probability for these electrons to penetrate the barrier. How the probability is affected if the barrier is doubled in width.

(d) Suppose we put a delta-function bump in the centre of the infinite square well: $H' = \alpha\delta\left(x - \frac{a}{2}\right)$, where α is a constant. Estimate both the first and second order correction to the allowed energies.

(e) Show that the expectation value of the potential energy in the n^{th} state of a linear harmonic oscillator is,

$$\langle V \rangle = \frac{1}{2}\hbar\omega\left(n + \frac{1}{2}\right)$$

(f) What is Schrodinger and Heisenberg picture in Quantum Mechanics? Derive Heisenberg equation of motion:

$$\frac{dA^H}{dt} = \frac{1}{i\hbar}[A^H, H]$$

where the symbols have their usual meanings.

3. Answer the following:(any four) **10 × 4 = 40**

(a) Following operator method, calculate the normalized ground state wave function and energy of a linear harmonic oscillator. 10

(b) Derive the uncertainty relation between two arbitrary operators using operator algebra. Hence derive the Heisenberg's uncertainty relation. 9+1

(c) Describe briefly the variational method. Use the variational method to estimate the ground state energy of the hydrogen atom. 2+8

P.T.O.

Hint: Use the trail wave function: $\psi(r, \theta, \phi) = e^{-r/\alpha}$

(d) If a perturbation of the form,

$$H' = \begin{cases} V_0, \; if \quad 0 < x < \frac{a}{2}, \quad and \quad 0 < y < \frac{a}{2}; \\ 0, \quad \text{otherwise} \end{cases}$$

is applied to a three-dimensional infinite cubical well. Find the first order correction to the energy of the ground state and triply degenerate first excited states. 10

(e) (i) Derive Klein Gordon equation in four dimensional covariant form. 5

(ii) In what way Klein Gordon equation is different from Schrodinger equation. Calculate the probability density provided by Klein Gordon equation and interpret the result. 5

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