2015

PHYSICS

Paper : 102 (Old Course)

CLASSICAL MECHANICS

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. For a system of N particles, the number of degrees of freedom is

    (a) N   (b) 2N   (c) 4N   (d) 3N

2. A non-holonomic constraint may be expressed in the form of

    (a) equality (b) inequality (c) vector (d) None of these

3. The equation of constraints in the case of particle moving on or outside the surface of a sphere of radius a is:

    (a) $x^2 + y^2 + z^2 \geq a^2$ (b) $x^2 + y^2 + z^2 \leq a^2$

    c) $x^2 = y^2 = z^2$ (d) $x^2 + y^2 + z^2 = 1$

(1)   *P.T.O.*

Answer all $2 \times 3 = 6$

4. Write down the conditions for a canonical transformation in terms of Poisson bracket.   2

5. What are holonomic and non-holonomic constraints?   2

6. Write down the Euler-Lagrange equation of motion for a continuous system.   2

Answer all $5 \times 4 = 20$

7. By using Lagrange equation, find the equation of motion of a compound pendulum in a vertical plane about a fixed horizontal axis.   5

8. Define Vorticity and obtain its equation of motion. $2 + 3 = 5$

9. For a pendulum hung from the ceiling of moving lift according to z(t), find the Lagrangian and Hamiltonian. $2 + 3 = 5$

10. Establish the Hamilton's equations in terms of Poisson bracket.   5

Answer any three from the following $9 \times 3 = 27$

11. State the Euler's theorem and prove the theorem. $3 + 6 = 9$

12. Prove that

$$[F,[G,K]] + [G,[K,F]] + [K,[F,G]] = 0 \qquad 9$$

13. State Navier – Stokes equation and obtain its approximation solution. $3 + 6 = 9$

14. State and establish Bernoulli's theorem. How the theorem is modified in case of steady irrotational flow? $2 + 4 + 3 = 9$

15. Explain the term generalized coordinate. Derive Lagrange's equation of motion from d'Alemberts principle. $2 + 7 = 9$

Answer any two from the following : $12 \times 2 = 24$

16. (a). Define infinitesimal canonical transformation. What is generating function? Find the expression for canonical transformation equation corresponding to the first two types of generating function. $3 + 3 + 6 = 12$

17. Derive Euler's dynamical equation of a rigid body. Discuss the force – free motion of a symmetrical top with its CG coinciding with the fixed point by Lagranges equations of motion. What is a 'symmetrical top'? $4 + 6 + 2 = 12$

18. (a). What do you mean by cyclic coordinate.   2

     (b). If the law of force acting on a planet is of the form

$$F(r) = -\frac{k}{r^3} \qquad (k > 0)$$

find the form of the orbit.   5

(c). Set up Hamilton's equations of motion for the Lagrangian

$$L(q, q, t) = m(q^2 \sin^2 \omega t + qq\omega \sin 2\omega t + q^2 \omega^2)/2 \qquad 5$$

19. What do you mean by chaos and strange attractors. Draw a bifurcation diagram to define the region of chaos.

$$4+4+4= 12$$

—— × ——