# 2018
## PHYSICS
## PHY 301
## MATHEMATICAL PHYSICS-II
### Full Marks: 80
### Time: 3 hours.

*The figures in the margin indicates full marks for the questions :*

1. Answer all the following multiple choice questions.          1x5=5

(a) An abelian group is a Group which satisfies the_____property.

   (i) Associative   (ii) Distributive   (iii) Identity   (iv) Commutative

(b) When the left and right cosets of a subgroup are equal, then the Group is

   (i) Abelian                    (ii) Non-abelian

   (iii) Homomorphic              (iv) Isomorphic.

(c) The Laplace transform of $e^{at}t^n$ is

   (i) $\dfrac{n}{s^{n+1}}$        (ii) $\dfrac{n!}{s^{n+1}}$

   (iii) $\dfrac{n}{(s-a)^{n+1}}$   (iv) $\dfrac{n!}{(s-a)^{n+1}}$

(d) The integral equation $y(x) = f(x) + \lambda \int_a^x K(x.t)y(t)dt$ is a

   (i) Volterra equation of first kind

   (ii) Volterra equation of second kind

   (iii) Fredholm equation of first kind

   (iv) Fredholm equation of second kind

(e) The number of generators in SU(3) group is

   (i) 1            (ii) 3            (iii) 8            (iv) None of these

**2.** Answer the following questions.  $2 \times 5 = 10$

**(a)** If $L[f(t)] = F(s)$, then show that
$$L[e^{at}f(t)] = F(s-a)$$

**(b)** If $F(s)$ is the Fourier transform of $f(x)$, then show that
$$\int_{-\infty}^{\infty} |f(x)|^2\, dx = \int_{-\infty}^{\infty} |F(s)|^2\, ds$$

**(c)** Find the Fourier transform of Dirac delta function

**(d)** What do you mean by cosets of a subgroup?

**(e)** Show that
$$\frac{\partial g_{ij}}{\partial x^k} = [ik,j] + [jk,i]$$

**3.** Answer the following questions (any five).  $5 \times 5 = 25$

**(a)** Solve the following partial differential equation by the method of separation of variables,  5
$$\frac{\partial^2 u}{\partial x \partial t} = e^{-t}\cos x$$
given $u = 0$, when $t = 0$ and $\frac{\partial u}{\partial t} = 0$, when $x = 0$.

**(b)** Find the expression of Laplace transform for periodic function. Show that Laplace transform of the wave form, $f(t) = \frac{2t}{3}$, if $0 \le t \le 3$ is  $3+2=5$
$$\frac{2e^{-3s}}{s(e^{-3s}-1)} + \frac{2}{3s^2}$$

**(c)** Show that the convolution of the functions $f(t) = e^{-t^2/\alpha^2}$ and $g(t) = e^{-t^2/\beta^2}$ is,  5
$$\alpha\beta \left[\frac{\pi}{\alpha^2 + \beta^2}\right]^{1/2} e^{-t^2/(\alpha^2+\beta^2)}$$

**(d)** Solve the following integral equation  5
$$y(x) = x + \lambda \int_0^1 (xt + t^2)y(t)dt$$

**(e)** Determine the components of the fundamental tensor in cylindrical coordinate system.  5

**(f)** Find the value of $g$ and $g^{ij}$, if $ds^2 = \frac{dr^2}{1 - \frac{r^2}{R^2}} + r^2(d\theta^2 + sin^2\theta\, d\phi^2)$, where $R$ is constant.  5

**4.** Answer the following questions (any four)  $10 \times 4 = 40$

**(a)** Derive transformation laws for the Christoffel symbols of the first kind and second kind.  10

**(b)** Solve the following equation  10
$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$
The boundary conditions are $u(x,0) = 3\sin n\pi x$, $u(0,t), u(l,t) = 0$, where $0 < x < l$.

**(c)** i. Use Laplace transform to solve the following differential equation
$$t\,y''(t) - ty'(t) + y(t) = 5$$  5
given, $y(0) = 5, y'(0) = 3$

ii.   Find the Fourier transform of the following function               5

$$f(x) = \begin{cases} 1, & if \ |x| < \dfrac{1}{2}a \\ 0, & if \ |x| > \dfrac{1}{2}a \end{cases}$$

**(d)** Solve the Fredholm integral equation of second kind using the method of separable kernel.                                               10

**(e)**   i.   What do you mean by class of a Group? Write the properties of classes.                                                           5

ii.   Given a Group $G = \{1, \omega, \omega^2\}$, where $\omega^3 = 1$, Using composition table prove that G is Abelian group.                   5