BU/EX/PG/SEM-1/101

2015

PHYSICS

Paper : 101

MATHEMATICAL PHYSICS

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer all the following multiple choice question

$1 \times 6 = 6$

(a) The value of $P_1(x)$ is

(i) 1 (ii) x

(iii) 0 (iv) $\frac{1}{2}(3x^2 - 1)$

(b) If A is a real symmetric matrix, the eigenvalues of A are

(i) Complex (ii) Real

(iii) of unit magnitude (iv) None of these

(c) The eigenvalues of a Hermitian matrix are

(i) Always zero (ii) Complex

(iii) Real (iv) All of these

(d) The ratio of $\frac{\Gamma(-3/2)}{\Gamma(3/2)}$ is

(i) π (ii) $\frac{\pi}{2}$ (iii) $\frac{8}{3}$ (iv) $\frac{3}{8}$

(e) If $J_o(x)$ and $J_1(x)$ are Bessel's functions then the expression for $J_1'(x)$ is

(i) $J_1'(x) = J_o(x) - \frac{1}{x}J_1(x)$ (ii) $J_1'(x) = J_o(x) + \frac{1}{x}J_1(x)$

(iii) $J_1'(x) = \frac{1}{x}J_o(x) - J_1(x)$ (iv) $J_1'(x) = \frac{1}{x}J_o(x) + J_1(x)$

(f) Eigen value of the matrix $A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

(i) $e^{\pm i\theta}$ (ii) $e^{\pm 2i\theta}$ (iii) $e^{\pm 3i\theta}$ (iv) $e^{\pm i\frac{\theta}{2}}$

2. Answer the following $2 \times 5 = 10$

(a) Establish the following recurrence relation

(i) $x J_n' = n J_n - x J_{n+1}$

(b) Show that

$$\int_0^1 \left(\log\frac{1}{y}\right)^{n-1} dy = \Gamma(n)$$

(c) What is harmonic function?

(d) What is unitary operator?

(e) Define the fundamental theorem of probability.

3. Answer the following (any five) $5 \times 5 = 25$

(a) Show the orthogonality of Legendre polynomial

$$\int_{-1}^{+1} P_m(x)P_n(x)dx = 0 \text{ (for } n = m)$$

(b) What is multi valued function? Give three examples.

(c) Show that $\beta(m,n) = \int_0^1 \frac{y^{m-1} + y^{n-1}}{(1+y)^{m+n}} dy$

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(d) Show that $\oint \frac{e^z}{\cos z} dz = -4\pi i \sinh(\pi/2)$ for $|z| = 3$

(e) Find the solution for Bessel's differential equation for indicial choice of $k = n$.

(f) Prove the following relation

$$\int_0^{\frac{\pi}{2}} \sin^p\theta \cos^q\theta \, d\theta = \frac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}{2\Gamma\left(\frac{p+q}{2}+1\right)}$$

4. Answer the following (any three) $9 \times 3 = 27$

(a) Show that Hermite polynomial is the solution of the differential equation

$$y'' - 2xy' + 2ny = 0$$

(b) (i) Verify whether the function $f(z) = e^{z^2}$ is analytic or not?

(ii) Find the function v such that $u + iv$ is analytic, where,

$$u = e^{-x}(x \sin y - y \cos y)$$

(c) Using Taylor's series expand $f(z) = \ln(1+z)$. What is the n[th] term of t function $f(z) = \sin z$ in the Taylor's series?

(d) Using the generating function of Hermite polynomial, find the expression of Hermite differential equation

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}$$

5. Answer (any one) $12 \times 1 = 12$

(a) Write the steps that followed to find the diagonal matrix of any square matrix. Diagonalize the following matrix A,

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

(b) Establish the hypergeometric function $F(\alpha, \beta, \gamma, x)$, represented by the hypergeometric series.