2015

PHYSICS

Paper : PHY 301

ADVANCED QUANTUM MECHANICS

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following $1 \times 5 = 5$

(a) Dirac - dalta function $\delta(\vec{r} - \vec{r}')$ can be written as - 1

(i) $\frac{1}{(2\pi)^3} \int e^{i\vec{p}\cdot(\vec{r}-\vec{r}')} d^3p$ (ii) $\frac{1}{(2\pi)^3} \int e^{-i\vec{p}\cdot(\vec{r}-\vec{r}')} d^3p$

(iii) $-\frac{1}{(2\pi)^3} \int e^{-i\vec{p}\cdot(\vec{r}-\vec{r}')} d^3p$ (iv) $-\frac{1}{(2\pi)^3} \int e^{i\vec{p}\cdot(\vec{r}-\vec{r}')} d^3p$

(b) Klein-Gordon equation is applicable only for 1

(i) Bosons (ii) Fermions (iii) Weakly interacting particles (iv) All of the above

(c) The partial wave analysis method is more appropriate for the low energy scattering phenomena. 1

(i) True (ii) False

(d) In scattering problem, the Born approximation is applicable only for a weak potential. 1

(i) True (ii) False

P.T.O.

(e) Dirac equation provides 1

(i) Positive energy solution

(ii) Negative energy solution

(iii) Both the positive & negative energy solution

(iv) None of the above

2. Answer the following (any five) $3 \times 5 = 15$

(a) Discuss the canonical quantization procedure to quantize a classical system. 3

Show that:

(b) $\gamma^\mu \gamma^\mu = 4$ 3

(c) $\gamma_\mu \not{A} \gamma^\mu = -2\not{A}$ 3

(d) Prove that $(\vec{\sigma} \cdot \vec{A})(\vec{\sigma} \cdot \vec{B}) = (\vec{A}.\vec{B}) + i\vec{\sigma}.(\vec{A} \times \vec{B})$, where the symbols have their usual meanings. 3

(e) Briefly discuss the Dirac hole theory. 3

(f) Establish Dirac equation in presence of electromagnetic field. 3

3. Answer the following (any three) $3 \times 5 = 15$

(a) Show how one can go from an 1-D discrete system to a continuous field system and derive the corresponding field equation. 5

(b) 650 MeV π^0 are scattered from a heavy and totally

absorbing nucleus of radius 1.4 fm.

(i) Estimate the total elastic and inelastic cross-sections.

(ii) Calculate the scattering amplitude and check the validity of the optical theorem. 2+3

(c) Using Dirac equation, obtain an expression for probability density and current density using the equation of continuity, 5

$$\frac{\partial \rho}{\partial t} + \nabla.J = 0$$

(d) Calculate the differential cross-section in the first Born approximation for a Coulomb potential $V(r) = z_1 z_2 \frac{e^2}{r}$, where z_1e and z_2e are the charges of the projectile and target particle respectively. 5

(e) Write short note on the schrödinger's Cat paradox. 5

4. Answer the following (any three) $3 \times 15 = 45$

(a) Quantize free electromagnetic field and arrive at the expressions for second quantized form of the field operator and the total energy operator. 10

(b) Why do we impose Coulomb (radiation) gauge in the above quantization? Why does one need to use transverse Kronecker delta in the ETCR of e.m. field in radiation gauge? 5

5. (a) Using partial wave analysis method prove the optical theorem:

$$\sigma t = \frac{4\pi}{k} \text{Im} f(0)$$

where the symbols have their usual meanings. 10

(b) Consider the elastic scattering of 100 MeV neutrons from a nucleus. The phase shifts measured in the experiment are $\delta_0 = 60^0$, $\delta_1 = 45^0$ and $\delta_2 = 30^0$ and all other phase shifts are negligible i.e. $\delta_l = 0$ for $l \geq 3$. Calculate the total scattering cross-section. 5

6. (a) Using Green's function method, derive the expression for differential scattering cross-section

$$\frac{d\sigma}{d\Omega} = \frac{1}{16\pi^2} \left| \int e^{-i\vec{k}.\vec{r}'} u(r')\psi(r') d^3r' \right|^2$$

where the symbols have their usual meanings. 10

(b) Using first order Born approximation, compute the expression for differential scattering cross-section in case of an elastic collision. Take the potential to be spherically symmetric. 5

7. (a) What do you mean by Lorentz covariance? Show that the Dirac equation is invariant under Lorentz transformation. 9

(b) Establish the following relation

$$\text{Tr}(\gamma^\mu \gamma^\nu \gamma^\alpha \gamma^\beta) = 4\left[g^{\mu\nu} g^{\alpha\beta} - g^{\mu\alpha} g^{\nu\beta} + g^{\mu\beta} g^{\nu\alpha}\right]$$ 6

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