Chapter 6

Conharmonically Flat Space with Dynamical Cosmological Term Λ in Five Dimensional Kaluza-Klein Space-Time

6.1 Introduction

Despite the numerous efforts made by several cosmological and astrophysical experts about the universe's future evolution in addition to their understanding of the present and past state of the universe, we are unable to make a definitive determination regarding the origin and evolution of the physical universe. Different cosmological observations (Perlmutter et al. (1999); Riess et al. (1998); Riess et al. (2004)) have confirmed that the universe is going through cosmic acceleration. It is believed that DE model plays a vital role in accelerating expansion of the universe. The most suitable representative for describing the DE model is the cosmological constant. Many prominent authors (Sahni and Starobinsky (2000); Padmanabhan (2003); Peebles and Ratra (2003); Padmanabhan (2008)) considered the variable cosmological constant as a time-varying parameter to explain the nature of dark energy problem. Al-Rawaf and Taha (1996) ; Al-Rawaf (1998) and Overduin and Cooperstock (1998) proposed a model of the universe with Λ term of the form $\Lambda = \beta\frac{\ddot{a}}{a}$, where β is a arbitrary constant.

Several researchers (Silveira and Waga (1994); Overduin (1999); Khadekar et al. (2008a); Sharif and Khanum (2011); Tiwari and Singh (2015); Tiwari (2016)) considered the dynemical cosmological term Λ in different contexts to study the cosmological model in general relativity.

In this chapter we discussed about the FRW type 5D Kaluza-Klein homogeneous and isotropic cosmological models by considering dynamical cosmological term as $\Lambda = \alpha H^2$ (where α is a constant and $H = \frac{\dot{a}}{a}$) to find out the solutions which are realistic with the observational facts.

6.2 The Metric and Field Equations:

We consider a homogeneous and isotropic FRW type 5D Kaluza-Klein space time is given by

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) + (1-kr^2)d\psi^2\right], \tag{6.1}$$

where $a(t)$ is a cosmic scale factor and $k = -1, 0, +1$ is the curvature parameter for open, flat and closed universe and the fifth coordinate ϕ is taken to be extended space like coordinate respectively.

The Einstein's field equation (with $c = G = 1$) may be written as

$$R_{ij} - \frac{1}{2}g_{ij}R = 8\pi T_{ij} - \Lambda g_{ij}, \tag{6.2}$$

where R_{ij} is the Ricci tensor, R is the Ricci scalar, g_{ij} is the metric tensor and T_{ij} is the energy momentum tensor of a perfect fluid given by

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij}, \tag{6.3}$$

The conharmonic curvature tensor for a relatives 5D space-time is

$$L^l_{ijk} = R^l_{ijk} - \frac{1}{3}(g_{ij}R^l_k - g_{ik}R^l_j + \delta^l_k R_{ij} - \delta^l_j R_{ik}), \tag{6.4}$$

For conharmonically flat space-time $L^l_{ijk} = 0$, we obtain

$$3R^l_{ijk} = g_{ij}R^l_k - g_{ik}R^l_j + \delta^l_k R_{ij} - \delta^l_j R_{ik}. \tag{6.5}$$

Contracting above equation with $j = l$ and taking summation over j, we obtain

$$R_{ik} = -\frac{1}{5}Rg_{ik}. \tag{6.6}$$

For conharmonically flat space-time, using the above equation the Einstein's field equation (6.2) reduce to

$$R_{ij} = \frac{16}{7}\pi T_{ij} - \frac{2}{7}\Lambda g_{ij}. \tag{6.7}$$

In co-moving coordinates system, for the flat $(k = 0)$ metric (6.4), the energy momentum tensor (6.3) and Einstein's field equation (6.7) reduces to

$$8\pi\rho - \Lambda = -14(\dot{H} + H^2) \tag{6.8}$$

$$8\pi p + \Lambda = \frac{7}{2}(\dot{H} + 4H^2) \tag{6.9}$$

where $H = \frac{\dot{a}}{a}$ is the Hubble's parameter and over head dot denotes the derivatives with respect to cosmic time t.

The EoS parameter (ω) which is considered as an important quantity in describing the dynamic of the universe in the ratio of the pressure (p) and the energy density (ρ) are given by

$$p = \omega\rho \tag{6.10}$$

Eliminating ρ from equations $(6.8) - (6.10)$ we obtain

$$\frac{7 + 28\omega}{2}\frac{\ddot{a}}{a} + \frac{21}{2}\frac{\dot{a}^2}{a^2} = (1 + \omega)\Lambda \tag{6.11}$$

which is the dynamical equation related to the scale factor a for the dynamical cosmological term Λ .

6.3 Solution of the Field Equations

In eqn. (6.11), involving three unknown parameters a, ω and Λ, so in order to obtain deterministic solution of the above equations we need one more physical equations involving these unknowns.

Here, we may consider the following value of Λ term (Silveira and Waga (1994); Overduin (1999); Khadekar et al. (2008a); Sharif and Khanum (2011); Tiwari and Singh (2015); Tiwari (2016))

$$\Lambda = \alpha \frac{\dot{a}^2}{a^2} \tag{6.12}$$

where α is an arbitrary constant.

Using eqn. (6.12) in eqn. (6.11) we obtain the general solution as

$$\frac{\ddot{a}}{a} + \frac{21 - 2(1+\omega)\alpha}{7+28\omega}\frac{\dot{a}^2}{a^2} = 0 \tag{6.13}$$

Integrating (6.13) we obtain

$$a(t) = [\frac{2(1+\omega)(14-\alpha)}{7(1+4\omega)}ct]^{\frac{7(1+4\omega)}{2(1+\omega)(14-\alpha)}} \tag{6.14}$$

Using eqn. (6.14) the model universe (6.1) becomes

$$ds^2 = dt^2 - [\frac{2(1+\omega)(14-\alpha)}{7(1+4\omega)}ct]^{\frac{7(1+4\omega)}{2(1+\omega)(14-\alpha)}} \left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) + (1-kr^2)d\psi^2\right], \tag{6.15}$$

The eqn. (6.15) represents 5D Kaluza-Klein cosmological model with the dynamical cosmological term Λ.

6.4 Physical Parameters of the Model and its Behaviour

For the cosmological model (6.15), the spatial volume (V), Hubble's parameter (H), expansion scalar (θ), deceleration parameter (q), energy density (ρ), pressure (p), cosmological constant (Λ) are obtained as follows:

$$V = [\frac{2(1+\omega)(14-\alpha)}{7(1+4\omega)}ct]^{\frac{14(1+4\omega)}{(1+\omega)(14-\alpha)}} \tag{6.16}$$

$$H = \frac{7(1+4\omega)}{2(1+\omega)(14-\alpha)t} \tag{6.17}$$

$$\theta = \frac{14(1+4\omega)}{(1+\omega)(14-\alpha)t} \tag{6.18}$$

$$q = -1 + \frac{2(1+\omega)(14-\alpha)}{7(1+4\omega)} \tag{6.19}$$

$$\rho = \frac{147(1+4\omega)}{32\pi(1+\omega)^2(14-\alpha)t^2} \tag{6.20}$$

$$P = \frac{147\omega(1+4\omega)}{32\pi(1+\omega)^2(14-\alpha)t^2} \tag{6.21}$$

$$\Lambda = \frac{49(1+4\omega)^2}{4(1+\omega)^2(14-\alpha)^2t^2} \tag{6.22}$$

Now we discuss scenarios for three types of physical acceptable universes ($\omega = 0, 1, \frac{1}{3}$):

6.4.1 Matter Dominated Solution (Cosmology for $\omega = 0$)

For $\omega = 0$ in this case we obtained the physical quantities as follows:

$$a(t) = [\frac{2(14-\alpha)}{7}ct]^{\frac{7}{2(14-\alpha)}} \tag{6.23}$$

$$V = [\frac{2(14-\alpha)}{7}ct]^{\frac{14}{(14-\alpha)}} \tag{6.24}$$

$$H = \frac{7}{2(14-\alpha)t} \tag{6.25}$$

$$\theta = \frac{14}{(14-\alpha)t} \tag{6.26}$$

$$q = -1 + \frac{2(14-\alpha)}{7} \tag{6.27}$$

$$\rho = \frac{147}{32\pi(14-\alpha)t^2} \tag{6.28}$$

$$P = 0 \tag{6.29}$$

$$\Lambda = \frac{49}{4(14-\alpha)^2 t^2} \tag{6.30}$$

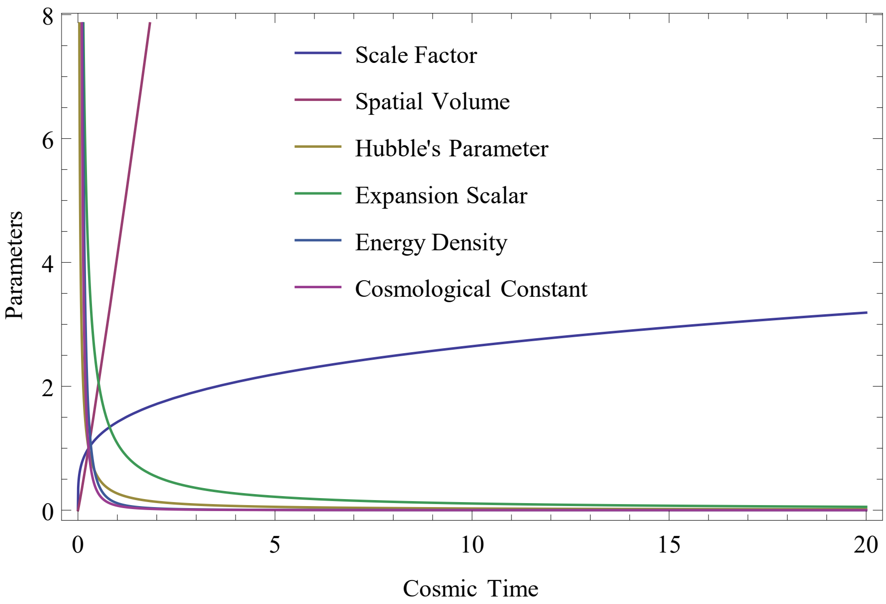

Figure 6.1: The plot of $a, V, H, \theta, \rho, \Lambda$ versus cosmic time t, for $\omega = 0, \alpha_1 = \alpha_2 = 1$.

6.4.2 Zeldovich Fluid Distribution (Cosmology for $\omega = 1$)

For $\omega = 1$ in this case the expressions for physical quantities are obtained as follows:

$$a(t) = [\frac{4(14-\alpha)}{35} ct]^{\frac{35}{4(14-\alpha)}} \tag{6.31}$$

$$V = [\frac{4(14-\alpha)}{35} ct]^{\frac{35}{(14-\alpha)}} \tag{6.32}$$

$$H = \frac{35}{4(14-\alpha)t} \tag{6.33}$$

$$\theta = \frac{35}{(14-\alpha)t} \tag{6.34}$$

$$q = -1 + \frac{4(14-\alpha)}{35} \tag{6.35}$$

$$\rho = p = \frac{735}{128\pi(14-\alpha)t^2} \tag{6.36}$$

$$\Lambda = \frac{1225}{16(14-\alpha)^2 t^2} \tag{6.37}$$

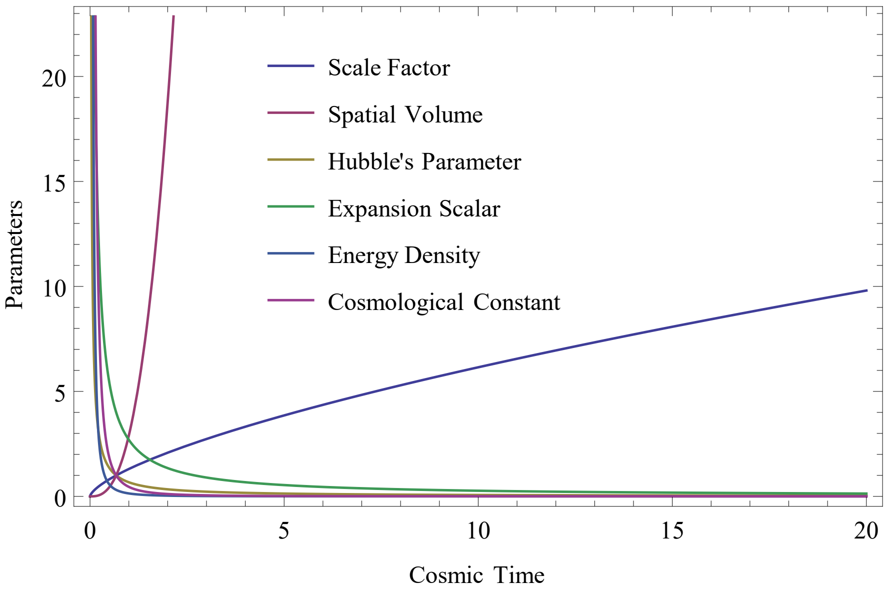

Figure 6.2: The plot of $a, V, H, \theta, \rho, \Lambda$ versus cosmic time t, for $\omega = 1, \alpha_1 = \alpha_2 = 1$.

6.4.3 Radiation Dominated Solution (Cosmology for $\omega = \frac{1}{3}$)

For $\omega = \frac{1}{3}$ the physical quantities are obtained as follows:

$$a(t) = [\frac{8(14-\alpha)}{49} ct]^{\frac{49}{8(14-\alpha)}} \tag{6.38}$$

$$V = [\frac{8(14-\alpha)}{49}ct]^{\frac{49}{2(14-\alpha)}} \tag{6.39}$$

$$H = \frac{49}{8(14-\alpha)t} \tag{6.40}$$

$$\theta = \frac{49}{2(14-\alpha)t} \tag{6.41}$$

$$q = -1 + \frac{8(14-\alpha)}{49} \tag{6.42}$$

$$\rho = \frac{3087}{512\pi(14-\alpha)t^2} \tag{6.43}$$

$$p = \frac{1029}{512\pi(14-\alpha)t^2} \tag{6.44}$$

$$\Lambda = \frac{2401}{64(14-\alpha)^2t^2} \tag{6.45}$$

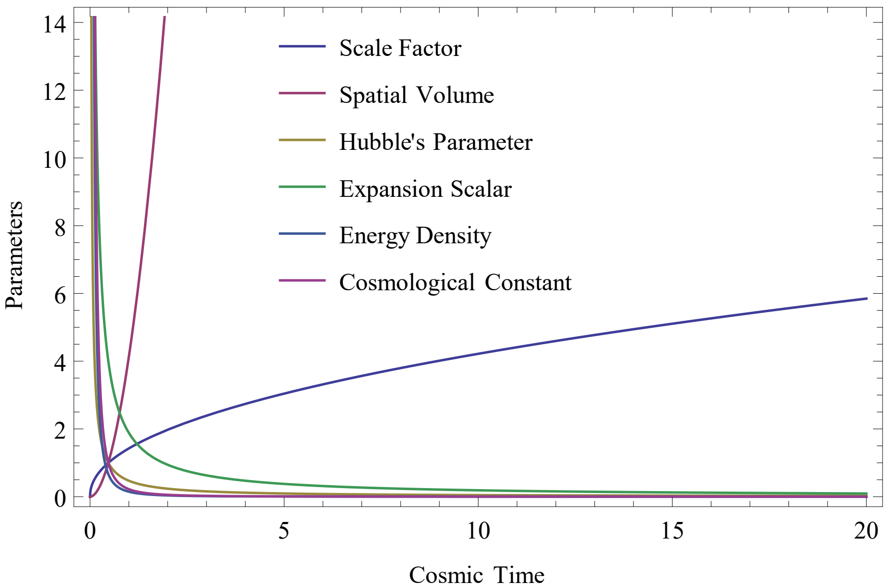

Figure 6.3: The plot of $a, V, H, \theta, \rho, \Lambda$ versus cosmic time t, for $\omega = \frac{1}{3}, \alpha_1 = \alpha_2 = 1$.

For the derived model, the statefinder parameter (r, s) can be written as

$$r = 1 - \frac{6(1+\omega)(14-\alpha)}{7(1+4\omega)} + \frac{8(1+\omega)^2(14-\alpha)^2}{49(1+4\omega)^2} \tag{6.46}$$

$$s = \frac{2(1+\omega)(14-\alpha)[35-28\omega-4\alpha(1+\omega)]}{21(1+4\omega)[4(1+\omega)(14-\alpha)-21(1+4\omega)]} \tag{6.47}$$

From the above eqns. (6.46) and (6.47), we observed that for $\alpha = 14$, the Statefinder parameter $(r, s) = (1, 0)$. Hence the derived model approached to ΛCDM model, which is good agreement with present day's observations (Ahmed and Pradhan (2014); Tiwari and Singh (2015); Goyal et al. (2019)).

Eqns. $(6.23) - (6.45)$ represents the cosmological parameters $a, V, H, \theta, q, p, \rho, \Lambda$ for the model (6.15). Figs. 6.1, 6.2 and 6.3 shows that the variation of parameters $(a, V, H, \theta, \rho, \Lambda)$ with respect to cosmic time t for all three types of physical acceptable models of the universes, $\omega = 0$ (matter dominated universe), $\omega = 1$ (Zeldovich universe), $\omega = \frac{1}{3}$ (radiation dominated universe) respectively.

From these figures we observed that the cosmic scale factor a and the spatial volume V both are increasing function of cosmic time t and tends to zero as $t \to 0$, which shows that the present model universe has a initial singularity (MacCallum (1971)).

The Hubble's parameter H and expansion scalar θ both are decreasing functions of cosmic time t. As $t \to \infty$ the Hubble's parameter and expansion scalar becomes zero, which totally agrees with the prevailing theories. Also we observed that $\frac{dH}{dt}$ is negative which indicate that the universe is expanding with an accelerated rate.

The variation of energy density ρ and cosmological constant Λ with respect to cosmic time t are depicted in figs. 6.1, 6.2 and 6.3 (for three different value of $\omega = 0, 1, \frac{1}{3}$). From these figures, we observed that ρ and Λ both are positive decreasing function of cosmic time t. It is seen that the energy density ρ and cosmological constant Λ are always remains positive ($\rho, \Lambda > 0$ for all $\alpha < 14$). For positive values of ρ and Λ, we have known that the physical universe is expanding with an accelerated rate. Initially when $t \to 0$ the energy density $\rho \to \infty$, which has a initial singularity.

From eqn. (6.19), we observed that the behavior of deceleration parameter $q < 0$ (for $\alpha > \frac{21}{2(1+\omega)}$), which indicates that the model universe is accelerating at present era, which agrees with present day's observational data (Riess et al. (1998); Garnavich et al. (1998); Schmidt et al. (1998); Perlmutter et al. (1999)).

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