Chapter 7

Five Dimensional Kaluza-Klein Cosmological Model in Conharmonically Flat Space with Hybrid Expansion Law

7.1 Introduction:

During the last two decades, many scientists and researchers are working very hard to comprehend the dynamics of the universe and its past, present, and future evolution. From different literatures and philosophical point of views we found that the present universe is expanding with an accelerated rate. In FRW type 5D Kaluza-Klein space-time, Einstein's theory of gravitation is considered to be a perfect theory to explain the expanding universe. Several cosmological models have been proposed by various authors in order to explain the unknown causes behind the acceleration of the expansion of the existing universe within the context of general theory of relativity. The 5D Kaluza-Klein cosmological model, which is spatially homogenous and isotropic, is very important in describe the behavior of the universe on a vast scale. The DE plays a crucial role for the accelerating of the universe. In order to describe the DE model, the cosmological constant is the most appropriate representative. The recent cosmological observations (Perlmutter et al. (1997); Perlmutter et al. (1998); Riess et al. (1998);

Garnavich et al. (1998); Schmidt et al. (1998); Perlmutter et al. (1999)) strongly suggests that the value of Λ is small and positive at the present era. Many eminent authors (Sahni and Starobinsky (2000); Padmanabhan (2003); Peebles and Ratra (2003); Padmanabhan (2008)) considered the variable cosmological constant to explain the nature of dark energy problem.

A conharmonic transformation is a conformal transformation that preserves the harmonicity of a function. Ishii (Ishii (1957)) studied the condition under which the harmonic functions remain invariant, who introduced the conharmonic transformation as a subgroup of the conformal transformations. In the context of general relativity, many authors (Ahsan and Siddiqui (2009); Siddiqui and Ahsan (2010); Tiwari and Singh (2015); Tiwari (2016); Tiwari and Shrivastava (2017); Goyal et al. (2019); Pradhan et al. (2020)) investigated the cosmological model of the universe in conharmonically flat space-time in the context of general relativity. Recently Dubey et al. (2021) discussed Sharma-Mittal holographic dark energy models in conharmonically flat space-time. By using the Hubble horizon as an IR cut-off and the deceleration parameter as a linear function of the Hubble's parameter, they explored the accelerated expansion of conharmonically flat space in regard to an isotropic and spatially homogeneous FRW universe through a newly proposed dark energy model called Sharma-Mittal holographic dark energy (SMHDE).

In this chapter, we wish to study FRW type 5D Kaluza-Klein cosmological models in conharmonically flat space with the help of hybrid expansion law for the average scale factor.

7.2 The Metric and Field Equations

We consider FRW type isotropic and homogeneous five dimensional Kaluza-Klein space-time given by

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) + (1-kr^2)d\psi^2\right], \tag{7.1}$$

where $a(t)$ is a cosmic scale factor and $k = -1, 0, +1$ is the curvature parameter for open, flat and closed universe and the fifth coordinate ϕ is taken to be extended space like coordinate respectively.

The Einstein's field equation (with $c = G = 1$) may be written as

$$R_{ij} - \frac{1}{2}g_{ij}R = 8\pi T_{ij} - \Lambda g_{ij}, \tag{7.2}$$

where R_{ij} is the Ricci tensor, R is the Ricci scalar, g_{ij} is the metric tensor and T_{ij} is the energy momentum tensor of a perfect fluid given by

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij}, \tag{7.3}$$

The conharmonic curvature tensor for a relatives five-dimensional space-time is

$$L^l_{ijk} = R^l_{ijk} - \frac{1}{3}(g_{ij}R^l_k - g_{ik}R^l_j + \delta^l_k R_{ij} - \delta^l_j R_{ik}), \tag{7.4}$$

For conharmonically flat space-time $L^l_{ijk} = 0$, we obtain

$$3R^l_{ijk} = g_{ij}R^l_k - g_{ik}R^l_j + \delta^l_k R_{ij} - \delta^l_j R_{ik}. \tag{7.5}$$

Contracting above equation with $j = l$ and taking summation over j, we obtain

$$R_{ik} = -\frac{1}{5}R g_{ik}. \tag{7.6}$$

For conharmonically flat space-time, using the above equation the Einstein's field equation (7.2) reduce to

$$R_{ij} = \frac{16}{7}\pi T_{ij} - \frac{2}{7}\Lambda g_{ij}. \tag{7.7}$$

In co-moving coordinates system, for the flat $(k = 0)$ metric (7.1) , the energy momentum tensor (7.3) and Einstein's field equation (7.7) reduces to

$$8\pi\rho - \Lambda = -14\frac{\ddot{a}}{a} \tag{7.8}$$

$$8\pi p + \Lambda = \frac{7}{2}(\frac{\ddot{a}}{a} + 3\frac{\dot{a}^2}{a^2}) \tag{7.9}$$

where over head dot denotes the derivatives with respect to cosmic time t.

7.3 Solutions of the Field Equations

Here, we have two independent field eqns. (7.8) and (7.9) connecting four unknowns viz. a, p, ρ and Λ.

So, first we considered the hybrid expansion law for the average scale factor proposed by Akarsu et al. (2014) and recently used by Reddy and Ramesh (2019).

$$a(t) = a_0 t^{\alpha_1} e^{\alpha_2 t} \tag{7.10}$$

where α_1, α_2 and a_0 are non-negative constant.

Secondly, the EoS parameter (ω) which is considered as an important quantity in describing the dynamics of the universe, is the ratio of the pressure (p) and the energy density (ρ) and is given by

$$p = \omega \rho \tag{7.11}$$

Now using eqn. (7.10) we write down the FRW type 5D Kaluza-Klein cosmological model as

$$ds^2 = dt^2 - a_0 t^{\alpha_1} e^{\alpha_2 t} \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) + (1 - kr^2) d\psi^2 \right], \tag{7.12}$$

The eqn. (7.12) represents FRW type 5D Kaluza-Klein cosmological model with the hybrid expansion law for the average scale factor.

For the cosmological model (7.12), the physical features of the cosmological parameters are obtained as follows:

The spatial volume (V) for the model is

$$V = (a_0 t^{\alpha_1} e^{\alpha_2 t})^4 \tag{7.13}$$

The average Hubble's parameter (H) is given by

$$H = \frac{\alpha_1}{t} + \alpha_2 \tag{7.14}$$

The expansion scalar (θ) is given by

$$\theta = 4(\frac{\alpha_1}{t} + \alpha_2) \tag{7.15}$$

The deceleration parameter (q) for the model is given by

$$q = -1 + \frac{\alpha_1}{(\alpha_1 + \alpha_2 t)^2} \tag{7.16}$$

The energy density (ρ) can be evaluated as

$$\rho = \frac{21\alpha_1}{2(1+\omega)t^2} \tag{7.17}$$

The pressure (p) as

$$p = \frac{21\omega\alpha_1}{2(1+\omega)t^2} \tag{7.18}$$

The cosmological constant (Λ) can be obtained as

$$\Lambda = 14(\frac{\alpha_1}{t} + \alpha_2)^2 - \frac{7}{2}\frac{(1+4\omega)\alpha_1}{(1+\omega)t^2} \tag{7.19}$$

For the derived model, the statefinder parameter (r, s) can be evaluated as

$$r = 1 - \frac{3\alpha_1}{(\alpha_1 + \alpha_2 t)^2} + \frac{2\alpha_1}{(\alpha_1 + \alpha_2 t)^3} \tag{7.20}$$

$$s = \frac{4\alpha_1}{6\alpha_1(\alpha_1 + \alpha_2 t) + 9(\alpha_1 + \alpha_2 t)^3} - \frac{2\alpha_1}{2\alpha_1 + 9(\alpha_1 + \alpha_2 t)^2} \tag{7.21}$$

7.4 Physical Interpretation of the Solutions:

The expression for Hubble's parameter (H) and expansion scalar (θ) are given by eqns (7.14) and (7.15), shows that the Hubble's parameter (H) and expansion scalar (θ) both are always positive and decreasing functions of cosmic time t. As $t \to 0$, both the Hubble's parameter (H) and expansion scalar (θ) infinite and as $t \to \infty$ both H and θ are approaching to finite value. The derived model universe has a point type initial singularity (MacCallum (1971)). Variation of H and θ with respect to cosmic time t is depicted in fig. (7.1). Also we observed that $\frac{dH}{dt}$ is negative which indicates that the universe is expanding with an accelerated rate.

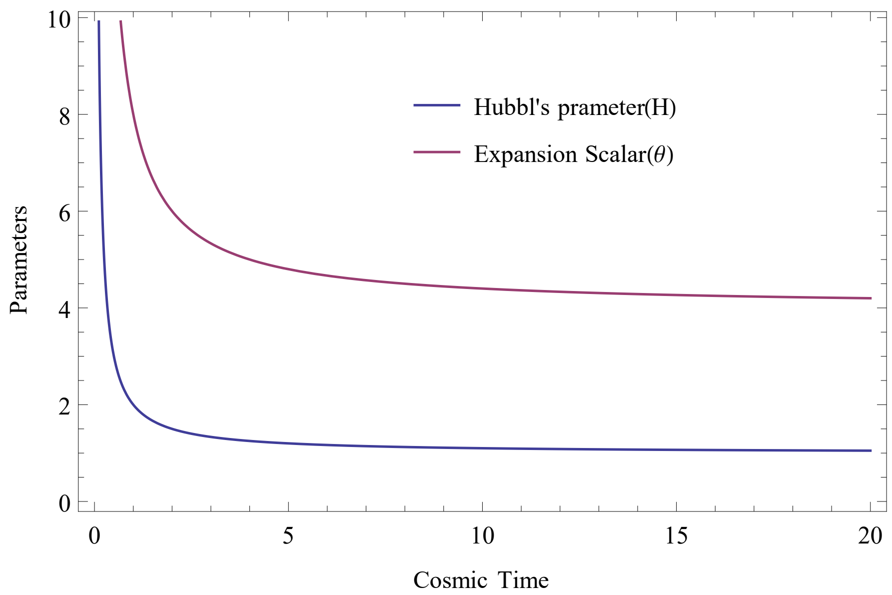

Figure 7.1: The plot of H and θ versus cosmic time t, for $\alpha_1 = \alpha_2 = 1$.

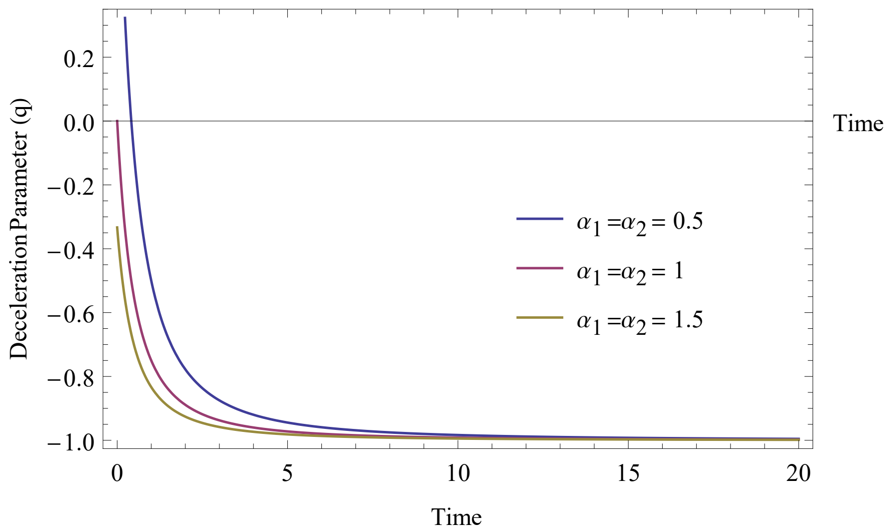

Figure 7.2: The plot of q versus cosmic time t.

From eqn. (7.13) it is observed that the spatial volume (V) is zero as $t = 0$ and it expands as time t increases and becomes infinity as $t \to \infty$, which represent the phenomenon of big bang.

Eqn. (7.16) represent the behavior deceleration parameter $q < 0$ for $t > \frac{\sqrt{\alpha_1}-\alpha_1}{\alpha_2}$, which suggestes that the model universe is in a accelerated phase of expansion, which agrees with present day's observations (Riess et al. (1998); Garnavich et al. (1998); Schmidt et al. (1998); Perlmutter et al. (1999)).

From eqns. (7.17) and (7.18) represents the expressions for energy density ρ and

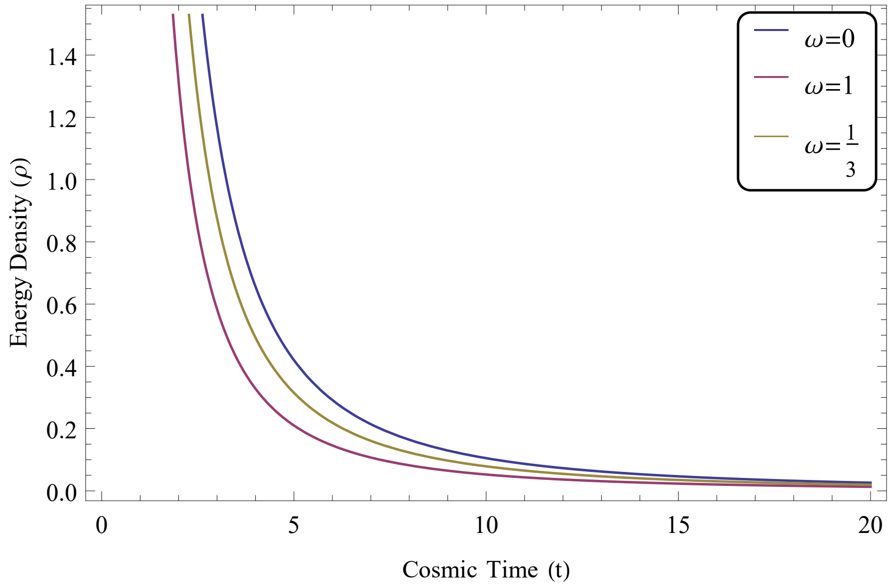

Figure 7.3: The plot of ρ versus cosmic time t, for $a_0 = \alpha_1 = \alpha_2 = 1$, $\omega = 0, 1, \frac{1}{3}$.

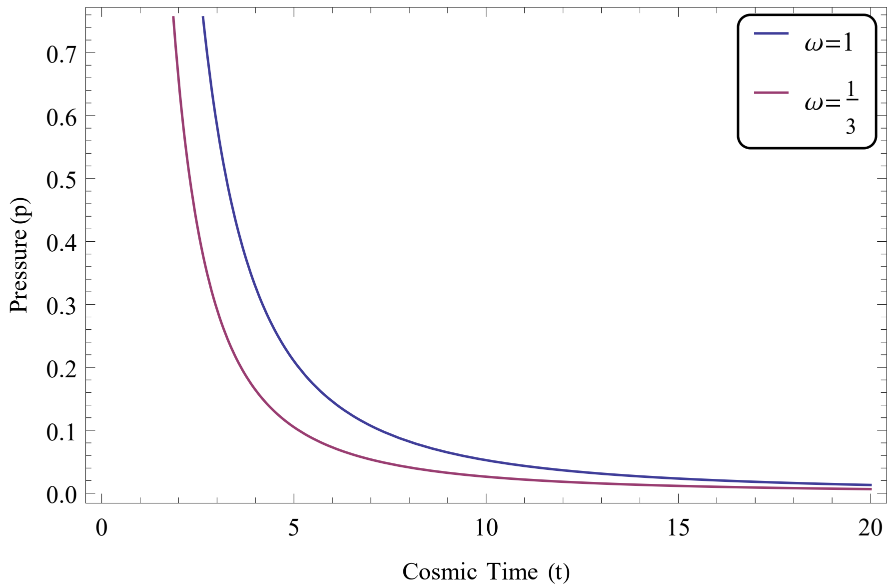

Figure 7.4: The plot of P versus cosmic time t, for $a_0 = \alpha_1 = \alpha_2 = 1$, $\omega = 0, 1, \frac{1}{3}$.

pressure p for the model (7.12). Figs. 7.3 and 7.4 shows the variation of ρ and p versus cosmic time t for all three types of models of universe, matter dominated universe $\omega = 0$, Zeldovich model $\omega = 1$ and radiating dominated universe $\omega = \frac{1}{3}$ respectively. From these figures it is seen that the energy density ρ and pressure p both are decreasing function of cosmic time t. As $t \to 0$, both ρ and p tends to infinity and becomes zero as $t \to \infty$, thus has an initial singularity.

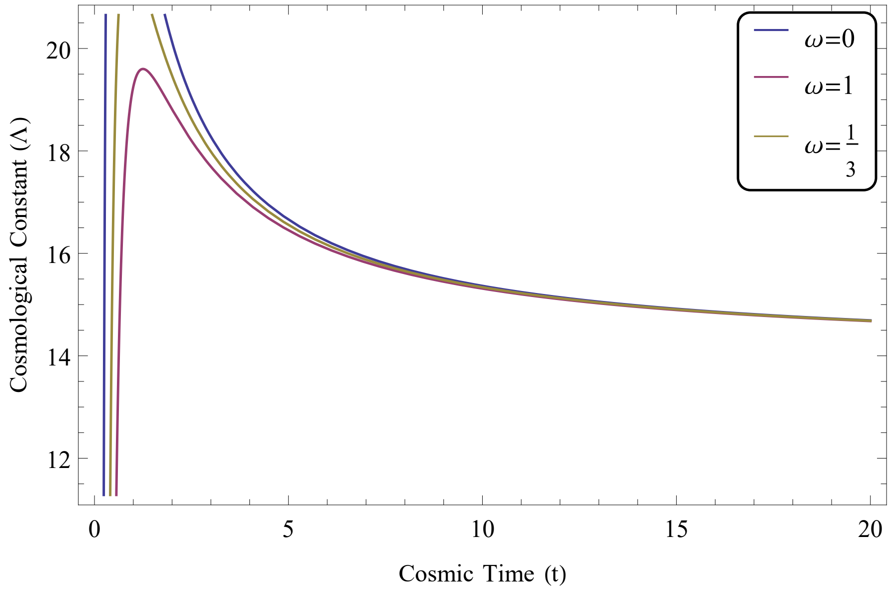

Figure 7.5: The plot of Λ versus cosmic time t, for $a_0 = \alpha_1 = \alpha_2 = 1$, $\omega = 0, 1, \frac{1}{3}$.

Eqn. (7.19) represents the expression for cosmological constant Λ for the model (7.12). Fig. 7.5 we have shown the variation of Λ versus cosmic time t for all three values of $\omega = 0, \omega = 1$ and $\omega = \frac{1}{3}$ correspondes to matter dominated universe, Zeldovich model and radiating dominated universe respectively. From this figure we observed that cosmological constant Λ is infinite as $t \rightarrow 0$ and tends to small positive value as $t \rightarrow \infty$, which agrees with accelerated expansion of the universe (Perlmutter et al. (1997); Perlmutter et al. (1998); Riess et al. (1998); Garnavich et al. (1998); Schmidt et al. (1998); Perlmutter et al. (1999)).

From eqns. (7.20) and (7.21), it is observed that the Statefinder parameter $(r, s) \rightarrow (1, 0)$ as $t \rightarrow \infty$. Hence the derived model approached to ΛCDM model which agrees with present day's observations (Ahmed and Pradhan (2014)); Tiwari and Singh (2015); Goyal et al. (2019)).