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Annexure 1

List of Publications

1. Pranjal Kumar Ray and Rajshekhar Roy Baruah (2021): **An Interacting and Non-interacting Two-Fluid Scenario for Dark Energy Models in Five Dimensional Kaluza-Klein Space-Time**, *Journal of Mathematical and Computational Science*, **11** (6): 7699-7716. https://doi.org/10.28919/jmcs/6382.

2. Pranjal Kumar Ray and Rajshekhar Roy Baruah (2022): **Anisotropic cloud string cosmological model with five-dimensional Kaluza-Klein space-time**, *Frontiers in Astronomy and Space Sciences*, **9**: 869020. https://doi.org/10.3389/fspas.2022.869020.

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AN INTERACTING AND NON-INTERACTING TWO-FLUID SCENARIO FOR DARK ENERGY MODELS IN FIVE DIMENSIONAL KALUZA-KLEIN SPACE TIME

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Abstract. In this paper, we studied the evaluation of dark energy parameter in the spatial homogeneous and anisotropic five dimensional Kaluza-Klein space time filled with barotropic fluid and dark energy. To solve the Einstein field equation by considering a variable deceleration parameter here we consider two cases; first, when these fluids are assumed to be not interacting each other and second, when they interact with each other. We have discussed the physical and geometrical importance of the two fluid scenario described in various aspects. We also discuss the jerk parameter and Statefinder parameter in our derived models which found that the model tends to the ΛCDM model.

Keywords: Kaluza-Klein space time; two-fluid; dark energy; variable deceleration parameter.

2010 AMS Subject Classification: 93A30.

1. Introduction

Cosmological observation shows that the universe is undergoing an accelerated expansion[1-4]. It is generally assumed that the dark energy is responsible for the acceleration of the universe [5-10]. So, in a late time accelerating and expanding of the universe has been confirmed from high red shift type Ia Supernovae experiment (SNe.Ia) [11-12], cosmic microwave background

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radiation (CMBR) [13], large scale structure [14], Sloan Digital Sky Survey (SDSS) [15, 16], First Year Wilkinson Microwave Anisotropy Probe(WMAP) [17] and Chandra X-ray observatory [18] combination strongly suggest the existence of the extra constituent in a universe with negative pressure called dark energy. Roughly dark energy occupies about 68.3%, dark matter occupies about 26.8% and baryonic matter occupies about 4.9% of the energy of our Universe [19].

The Kaluza-Klein[20,21] theories have exposed how the gravity and the electromagnetism can be unified from Einstein's field equations generalized to five dimensions. In a certain sense the Kaluza-Klein theory resembles ordinary gravity, except that it is inscribed in five dimensions instead of four. This theory has been regarded as a candidate of fundamental theory due to the possible work of unifying the fundamental principle. Different authors study the five dimensional Kaluza-Klein space-time viz. Chodos et al. [22], Appelquist et al. [23] studied some cosmological models in five dimensional Kaluza-Klein space-time. D. R. K. Reddy et al. [24], R. L. Naidu et al. [25], Reddy et al. [26] , T.Ramprasad et al.[27] investigated five dimensional Kaluza-Klein cosmological model in the framework of Brans-Dicke theory of gravitation. G. S. Khadekar [28] , Namrata I. Jain et al. [29] investigated viscous cosmological model in five dimensional Kaluza-Klein space time. G.S. Khadekar et al. [30] and M. Sharif et al. [31] discussed behavior of gravitational and cosmological term in five dimensional Kaluza-Klein space time. R.Venkateswarlu, K. P. Kumar [32], G. S. Khadekar et al.[33], G. C. Samanta and S. N. Dhal [34] investigated about the higher dimensional FRW cosmological models. H. Amirhashchi et al. [35], B. Saha et al. [36], H. Amirhashchi et al. [37], A. Pradhan et al. [38], V.B.Raut et al. [39] had studied various aspects in dark energy models in FRW universe. Many authors works on FRW universe mentioned above are the motivation behind this work on accelerating and ΛCDM (Lambda cold dark matter) model.

In this paper, we have investigated the evaluation of dark energy parameter in the spatial homogeneous and anisotropic five dimensional Kaluza-Klein space time filled with barotropic fluid and dark energy. To solve the Einstein field equation by considering a variable deceleration parameter $q = -\frac{a\ddot{a}}{\dot{a}^2} = \beta H + \alpha$. In Section 1, we discussed the introduction of Kaluza-Klein cosmology. The Kaluza-Klein models and its field equations are presented in Section 2 and 3.

In Section 4 and 5, we discussed the interacting and non-interacting two fluid model. Section 6, we discussed the jerk parameter and Statefinder parameters in our derived models. In Section 7, we discussed the physical interpretation of our model. Finally, conclusions and summarized are given in the last section 8.

2. METRIC AND FIELD EQUATIONS

We consider the special homogeneous and anisotropic five-dimensional Kaluza-Klein space time given by

$$ds^2 = dt^2 - A^2(dx^2 + dy^2 + dz^2) - B^2 d\varphi^2 \tag{1}$$

where A and B are function of cosmic time t only.

Einstein's field equation (with $8\pi G = 1$ and $c = 1$) is given by

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij} \tag{2}$$

where R_{ij} is the Ricci tensor R is the Ricci scalar g_{ij} is the metric tensor and T_{ij} is the energy momentum tensor of a perfect fluid and dark energy respectively.

The energy momentum tensor for two fluid is given by

$$T_{ij} = T_{ij}^m + T_{ij}^D \tag{3}$$

where

$$T_{ij}^m = (\rho_m + p_m)]\mu_i\mu_j - p_m g_{ij} \tag{4}$$

$$T_{ij}^D = (\rho_D + p_D)\mu_i\mu_j - p_D g_{ij} \tag{5}$$

here $\mu^i\mu_i = 1, \mu^i\mu_j = 0$ and ρ_m and p_m are the energy density and pressure of the perfect fluid and ρ_D and p_D are the energy density and pressure of the dark energy component.

The equation of state parameter (ω) which is consider as an important quantity in describing the dynamic of the universe in the ratio of the pressure (p) and the energy density (ρ) are given by

$$p_m = \omega_m \rho_m \tag{6}$$

and

$$p_D = \omega_D \rho_D \tag{7}$$

The important physical parameter like spatial volume(V), Hubble parameter(H), expansion scalar (θ), shear scalar (σ^2), anisotropic parameter (Δ) and deceleration parameter (q) for the metric (1) are defined as

$$V = a^4 = A^3B \tag{8}$$

$$H = \frac{1}{4}(\frac{3\dot{A}}{A} + \frac{\dot{B}}{B}) \tag{9}$$

$$\theta = 4H = \frac{3\dot{A}}{A} + \frac{\dot{B}}{B} \tag{10}$$

$$\sigma^2 = \frac{1}{2}[\sum_{i=1}^{4} H_i^2 - 4H^2] = \frac{4}{2}\Delta H^2 \tag{11}$$

$$\Delta = \frac{1}{4}\sum_{i=1}^{4}(\frac{\Delta H_i}{H})^2 \tag{12}$$

where $\Delta H_i = H_i - H$

and

(13) $$q = -\frac{\ddot{a}a}{\dot{a}^2} = \frac{d}{dt}\frac{1}{H} - 1$$

where an overhead dot denote derivative with respect to cosmic time t.

The Einstein field equation (2) and (3) for the metric (1) it follows that

(14) $$2\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + 2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}^2}{A^2} = -(p_m + p_D)$$

(15) $$3\frac{\ddot{A}}{A} + 3\frac{\dot{A}^2}{A^2} = -(p_m + p_D)$$

(16) $$3\frac{\dot{A}^2}{A^2} + 3\frac{\dot{A}\dot{B}}{AB} = \rho_m + \rho_D$$

An over dot indicate a derivatives with respect to cosmic time t.

The energy conservation equation ($T^{ij}_{;j} = 0$) which yields

(17) $$\dot{\rho} + 4H(\rho + p) = 0$$

where $p = p_m + p_D$ and $\rho = \rho_m + \rho_D$.

3. Solution of the Field Equations

We consider deceleration parameter (q) as a linear function of hubble parameter[40-42]:

(18) $$q = -\frac{a\ddot{a}}{\dot{a}^2} = \beta H + \alpha$$

Here α and β arbitrary constants.

we take $\alpha = -1$ in equation (18)

(19) $$q = -\frac{a\ddot{a}}{\dot{a}^2} = -1 + \beta H$$

which yields the following differential equation

$$\frac{a\ddot{a}}{\dot{a}^2}+\beta\frac{\dot{a}}{a}-1=0 \tag{20}$$

which on integration gives

$$a(t)=exp\left[\frac{1}{\beta}\sqrt{2\beta t+c}\right]$$

$$=e^{\frac{1}{\beta}\sqrt{2\beta t+c}} \tag{21}$$

where c is an integrating constant.

Subtracting equation (14) from equation (15) we get,

$$\frac{d}{dt}(\frac{\dot{A}}{A}-\frac{\dot{B}}{B})+(\frac{\dot{A}}{A}-\frac{\dot{B}}{B})\frac{\dot{V}}{V}=0$$

which on integrating gives

$$\frac{\dot{A}}{A}-\frac{\dot{B}}{B}=\frac{\lambda}{V} \tag{22}$$

where λ is an integration constant.

Using equation (21) in equation (22) and then integrating we get the scale factors are

$$A(t)=e^{\frac{1}{\beta}\sqrt{2\beta t+c}}exp[-\frac{\lambda\beta}{64}e^{\frac{-4}{\beta}\sqrt{2\beta t+c}}(\frac{4}{\beta}\sqrt{2\beta t+c}+1)] \tag{23}$$

and

$$B(t)=e^{\frac{1}{\beta}\sqrt{2\beta t+c}}exp[\frac{3\lambda\beta}{64}e^{\frac{-4}{\beta}\sqrt{2\beta t+c}}(\frac{4}{\beta}\sqrt{2\beta t+c}+1)] \tag{24}$$

Therefore the metric (1) reduce to

$$ds^2 = dt^2 - e^{\frac{1}{\beta}\sqrt{2\beta t+c}} exp[-\frac{\lambda\beta}{64} e^{\frac{-4}{\beta}\sqrt{2\beta t+c}} (\frac{4}{\beta}\sqrt{2\beta t+c}+1)](dx^2+dy^2+dz^2) - e^{\frac{1}{\beta}\sqrt{2\beta t+c}} exp[\frac{3\lambda\beta}{64} e^{\frac{-4}{\beta}\sqrt{2\beta t+c}} (\frac{4}{\beta}\sqrt{2\beta t+c}+1)] d\phi^2 \quad (25)$$

The physical parameters of the model are given by

$$V = e^{\frac{4}{\beta}\sqrt{2\beta t+c}} \quad (26)$$

$$H = \frac{1}{\sqrt{2\beta t+c}} \quad (27)$$

$$\theta = 4H = \frac{4}{\sqrt{2\beta t+c}} \quad (28)$$

$$\therefore \Delta = \frac{3\lambda^2}{16}(2\beta t+c) e^{\frac{-8}{\beta}\sqrt{2\beta t+c}} \quad (29)$$

$$\sigma^2 = \frac{3\lambda^2}{32} e^{\frac{-8}{\beta}\sqrt{2\beta t+c}} \quad (30)$$

$$q = -1 + \frac{\beta}{\sqrt{2\beta t+c}} \quad (31)$$

In the following sections we deal with two cases (i) Non-interacting two fluid model and (ii) Interacting two fluid model.

4. NON-INTERECTING TWO-FLUID MODEL

The conservation equation for the dark and barotropic fluid separately

(32) $$\dot{\rho}_m + 4H(p_m + \rho_m) = 0$$

(33) $$\dot{\rho}_D + 4H(p_D + \rho_D) = 0$$

Integrating (32) we get,

(34) $$\rho_m = \rho_0 e^{\frac{-4(1+\omega_m)}{\beta}\sqrt{2\beta t + c}}$$

where ρ_0 is an integrating constant.

Using (23), (24) and (34) in equation (15) and (16) we obtain

(35) $$P_D = -3\left[\frac{2 + \frac{\lambda^2}{8}(2\beta t + c)e^{\frac{-8}{\beta}\sqrt{2\beta t + c}} - \beta(2\beta t + c)^{\frac{-1}{2}}}{(2\beta t + c)}\right] - \omega_m \rho_0 e^{\frac{-4(1+\omega_m)}{\beta}\sqrt{2\beta t + c}}$$

(36) $$\rho_D = 3\left[\frac{2 - \frac{\lambda^2}{8}(2\beta t + c)e^{\frac{-8}{\beta}\sqrt{2\beta t + c}}}{(2\beta t + c)}\right] - \rho_0 e^{\frac{-4(1+\omega_m)}{\beta}\sqrt{2\beta t + c}}$$

Using equations (35) and (36) we can find the expression for EOS of dark filed in terms of time as

$$\omega_D = \frac{P_D}{\rho_D}$$

(37)
$$\omega_D = -\left[\frac{6 + \frac{3\lambda^2}{8}(2\beta t + c)e^{\frac{-8}{\beta}\sqrt{2\beta t + c}} - 3\beta(2\beta t + c)^{\frac{-1}{2}} + \omega_m \rho_0 (2\beta t + c)e^{\frac{-4(1+\omega_m)}{\beta}\sqrt{2\beta t + c}}}{6 - \frac{3\lambda^2}{8}(2\beta t + c)e^{\frac{-8}{\beta}\sqrt{2\beta t + c}} - \rho_0(2\beta t + c)e^{\frac{-4(1+\omega_m)}{\beta}\sqrt{2\beta t + c}}}\right]$$

The expressions of matter-energy-density parameter (Ω_m) and dark energy density parameter (Ω_D) are given by

$$\Omega_m = \frac{\rho_m}{6H^2} = \frac{\rho_0 e^{\frac{-4(1+\omega_m)}{\beta}\sqrt{2\beta t+c}}}{6(2\beta t+c)^{-1}} \tag{38}$$

and

$$\Omega_D = \frac{\rho_D}{6H^2} = 1 - \frac{\lambda^2}{16}(2\beta t+c)e^{\frac{-8}{\beta}\sqrt{2\beta t+c}} - \frac{1}{6}\rho_0(2\beta t+c)e^{\frac{-4(1+\omega_m)}{\beta}\sqrt{2\beta t+c}} \tag{39}$$

The total energy density (Ω) is given by

$$\Omega = \Omega_m + \Omega_D = 1 - \frac{\lambda^2}{16}(2\beta t+c)e^{\frac{-8}{\beta}\sqrt{2\beta t+c}} \tag{40}$$

5. INTERECTING TWO-FLUID MODEL

In this section, we can write the energy conservation equation for the dark and barotropic fluid as

$$\dot{\rho}_m + 4(1+\omega_m)\rho_m H = Q \tag{41}$$

$$\dot{\rho}_D + 4(1+\omega_D)\rho_D H = -Q \tag{42}$$

where the quantity Q represents the interecting between the componants of matter and dark energy.We consider $Q > 0$, this ensures that the energy is being transferred from dark energy to the matter component.

We consider

$$Q = 4Hk\rho_m \tag{43}$$

where k is coupling constant.

Using (43) in equation (41) we get

$$\rho_m = \rho_0 e^{\frac{-4(1+\omega_m-k)}{\beta}\sqrt{2\beta t+c}} \tag{44}$$

Now we again obtain P_D and ρ_D.

$$P_D = -3\left[\frac{2+\frac{\lambda^2}{8}(2\beta t+c)e^{\frac{-8}{\beta}\sqrt{2\beta t+c}} - \beta(2\beta t+c)^{\frac{-1}{2}}}{(2\beta t+c)}\right] - \omega_m\rho_0 e^{\frac{-4(1+\omega_m-k)}{\beta}\sqrt{2\beta t+c}} \tag{45}$$

$$\rho_D = 3\left[\frac{2-\frac{\lambda^2}{8}(2\beta t+c)e^{\frac{-8}{\beta}\sqrt{2\beta t+c}}}{(2\beta t+c)}\right] - \rho_0 e^{\frac{-4(1+\omega_m-k)}{\beta}\sqrt{2\beta t+c}} \tag{46}$$

Also the equation of state parameter (ω_D) is obtained as

(47)

$$\omega_D = -\left[\frac{6+\frac{3\lambda^2}{8}(2\beta t+c)e^{\frac{-8}{\beta}\sqrt{2\beta t+c}} - 3\beta(2\beta t+c)^{\frac{-1}{2}} + \omega_m\rho_0(2\beta t+c)e^{\frac{-4(1+\omega_m-k)}{\beta}\sqrt{2\beta t+c}}}{6-\frac{3\lambda^2}{8}(2\beta t+c)e^{\frac{-8}{\beta}\sqrt{2\beta t+c}} - \rho_0(2\beta t+c)e^{\frac{-4(1+\omega_m-k)}{\beta}\sqrt{2\beta t+c}}}\right]$$

The expression of matter energy density-parameter (Ω_m) and dark energy density parameter (Ω_D) are given as

$$\Omega_m = \frac{\rho_m}{6H^2} = \frac{\rho_0 e^{\frac{-4(1+\omega_m-k)}{\beta}\sqrt{2\beta t+c_1}}}{6(2\beta t+c_1)^{-1}} \tag{48}$$

and

$$\Omega_D = \frac{\rho_D}{6H^2} = 1 - \frac{\lambda^2}{16}(2\beta t+c)e^{\frac{-8}{\beta}\sqrt{2\beta t+c}} - \frac{1}{6}\rho_0(2\beta t+c)e^{\frac{-4(1+\omega_m-k)}{\beta}\sqrt{2\beta t+c}} \tag{49}$$

The total energy density (Ω) is given by

$$\Omega = \Omega_m + \Omega_D = 1 - \frac{\lambda^2}{16}(2\beta t+c)e^{\frac{-8}{\beta}\sqrt{2\beta t+c}} \tag{50}$$

6. The Jerk Parameter (j) and Statefinder parameter (r,s)

The Jerk Parameter in cosmology is defined as the dimensionless third derivative of the scale factor with respect to cosmic time t [43,44]. It is defined as

(51) $$j(t) = \frac{\dddot{a}}{aH^3} = \frac{(a^2H^2)''}{2H^2}$$

where over dots and primes denote derivatives with respect to cosmic time and the scale factor respectively.

The Jerk Parameter (j) appears in the fourth term of a Taylor expansion of the scale factor around a_0.

(52) $$\frac{a(t)}{a_0} = 1 + H_0(t-t_0) - \frac{1}{2}q_0H_0^2(t-t_0)^2 + \frac{1}{6}j_0H_0^3(t-t_0)^3 + 0\left[(t-t_0)^4\right]$$

in equation (37) we can be written as

(53) $$j(t) = q + 2q^2 - \frac{\dot{q}}{H}$$

From equation (27) and (31) we reduce to

(54) $$j(t) = 1 - \frac{3\beta}{\sqrt{2\beta t + c}} + \frac{3\beta^2}{2\beta t + c}$$

For flat ΛCDM model, the jerk parameter (j) has the value $j = 1$.

The Statefinder parameter (r,s) [45] defined as follows

(55) $$r = \frac{\dddot{a}}{aH^3} = 1 + 3\frac{\dot{H}}{H^2} + \frac{\ddot{H}}{H^3}$$

and

(56) $$s = \frac{r-1}{3(q-\frac{1}{2})}$$

From equations $(27),(31),(55)$ and (56) we obtained

$$r = 1 - \frac{3\beta}{\sqrt{2\beta t + c}} + \frac{3\beta^2}{2\beta t + c} \tag{57}$$

and

$$s = \frac{-2\beta\sqrt{2\beta t + c} + 2\beta^2}{2\beta\sqrt{2\beta t + c} - 3(2\beta t + c)} \tag{58}$$

From the above result it is observed that as $t \to \infty$, $(r,s) \to (1,0)$ which gives that the model tends to the ΛCDM model as in recent observation[45].

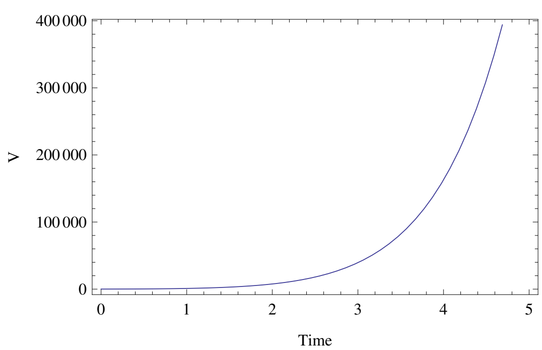

FIGURE 1. The plot of spatial volume V versus cosmic time t, for $\beta = 1$ and $c = 1$.

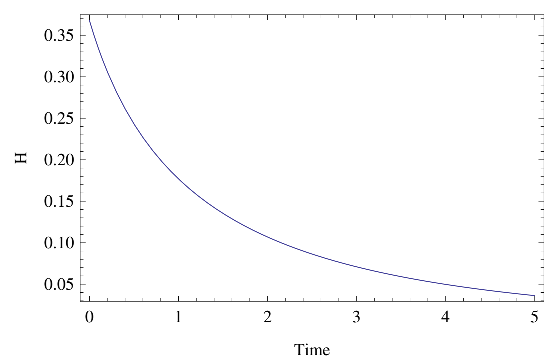

FIGURE 2. The plot of hubble parameter H versus cosmic time t, for $\beta = 1$ and $c = 1$.

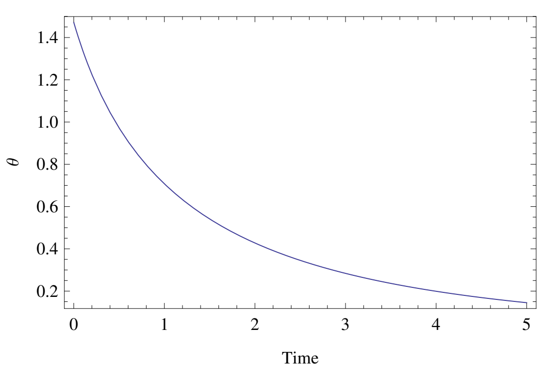

FIGURE 3. The plot of expansion scalar θ versus cosmic time t,for $\beta = 1$ and $c = 1$.

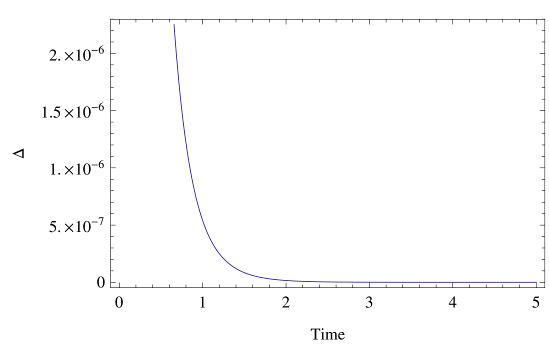

FIGURE 4. The plot of anisotropic parameter Δ versus cosmic time t,for $\beta = 1$, $\lambda = 1$ and $c = 1$.

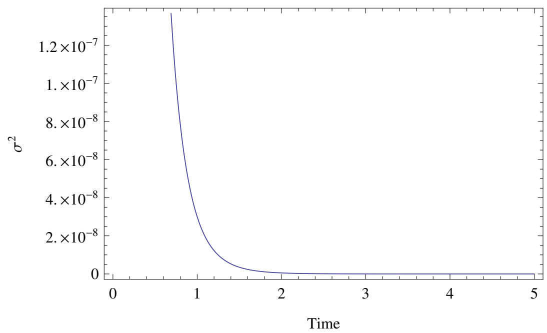

FIGURE 5. The plot of shear scalar σ^2 versus cosmic time t,for $\beta = 1$, $\lambda = 1$ and $c = 1$.

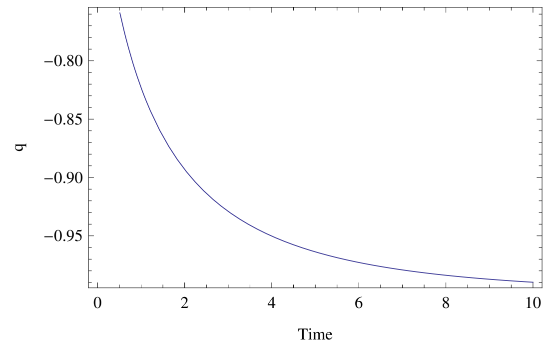

FIGURE 6. The plot of deceleration parameter q versus cosmic time t,for $\beta = 1$ and $c = 1$.

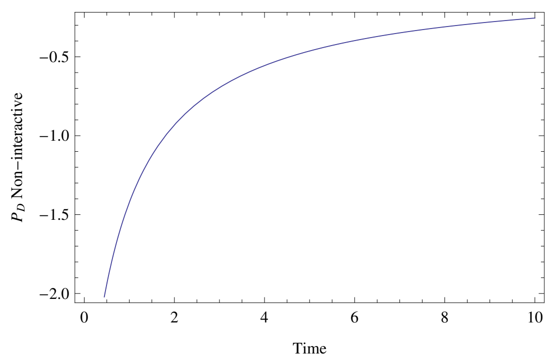

FIGURE 7. The plot of pressure p_D versus cosmic time t for $\beta = 1, c = 1, \lambda = 1$, $\rho_0 = 1$ and $\omega_m = 0.5$ for non-interactive case.

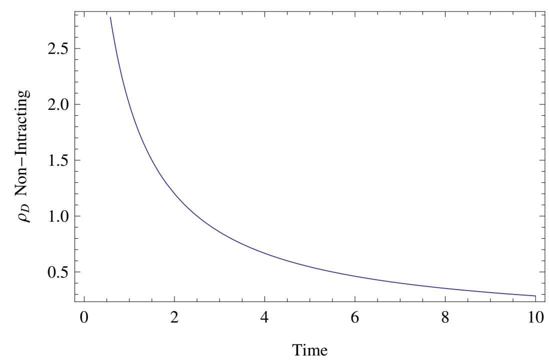

FIGURE 8. The plot of energy density ρ_D versus cosmic time t for $\beta = 1, c = 1, \lambda = 1$, $\rho_0 = 1$ and $\omega_m = 0.5$ for non-interactive case.

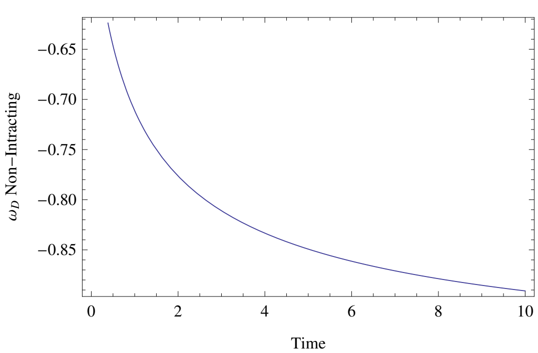

FIGURE 9. The plot of EoS parameter ω_D versus cosmic time t for $\beta = 1, c = 1, \lambda = 1$, $\rho_0 = 1$ and $\omega_m = 0.5$ for non-interactive case.

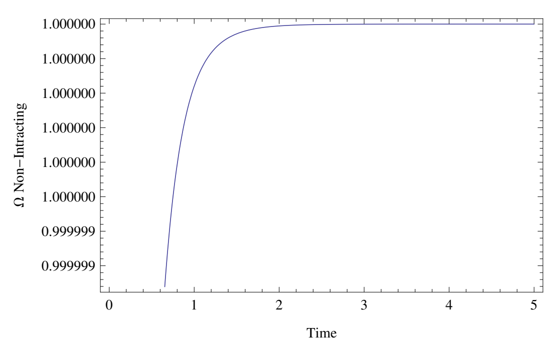

FIGURE 10. The plot of total energy density Ω versus cosmic time t for $\beta = 1, c = 1, \lambda = 1$, $\rho_0 = 1$ and $\omega_m = 0.5$ for non-interactive case.

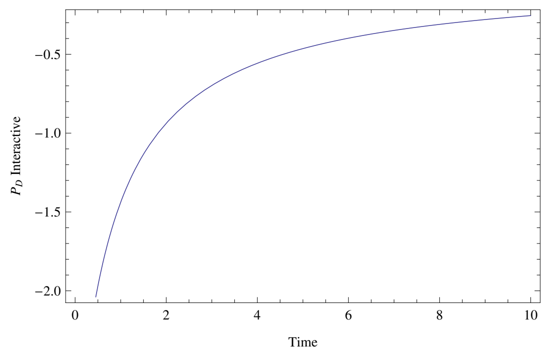

FIGURE 11. The plot of pressure p_D versus cosmic time t for $\beta = 1, c = 1, \lambda = 1$, $\rho_0 = 1$ and $\omega_m = 0.5$ for interactive case.

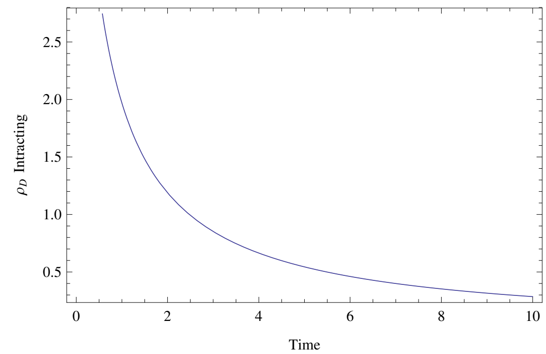

FIGURE 12. The plot of energy density ρ_D versus cosmic time t for $\beta = 1, c = 1, \lambda = 1$, $\rho_0 = 1$ and $\omega_m = 0.5$ for interactive case.

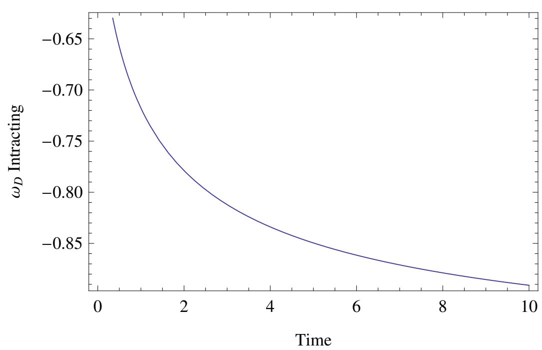

FIGURE 13. The plot of EoS parameter ω_D versus cosmic time t for $\beta = 1, c = 1, \lambda = 1$, $\rho_0 = 1$ and $\omega_m = 0.5$ for interactive case.

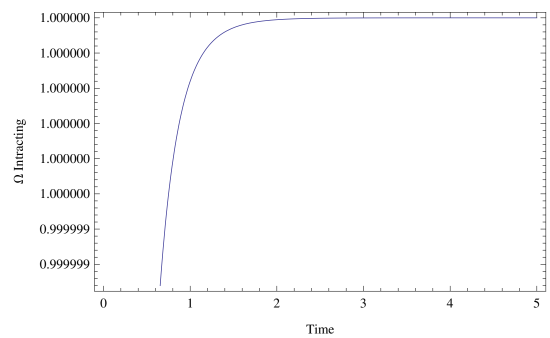

FIGURE 14. The plot of total energy density Ω versus cosmic time t for $\beta = 1, c = 1, \lambda = 1$, $\rho_0 = 1$ and $\omega_m = 0.5$ for interactive case.

7. PHYSICAL INTERPRETATION

Figure 1 depicts the spatial volume V versus cosmic time t. It is observed that the behavior of spatial volume V is zero as cosmic time $t = 0$ and increase as t tends to infinity. In figures 2 and 3 shows that both Hubble parameter H and expansion scalar θ positive, decrease with cosmic time t and tends to zero as $t \to \infty$. Figures 4 and 5 shown that the anisotropic parameter Δ and shear scalar σ^2 tends to zero as cosmic time $t \to \infty$. From figure 6 corresponding to the equation (31) it is shows that the deceleration parameter q is decreasing function of cosmic time t. It is observed that $q > 0$ for $t < \frac{\beta^2 - c}{2\beta}$ which indicates that the universe is a decelerating phase and $q < 0$ for $t > \frac{\beta^2 - c}{2\beta}$ which indicates that the universe is a accelerating phase, which is in the good agreement with the recent observations.

Figures 7 and 11 shows that the pressure p_D for both non-interacting and interacting cases it is observed that pressure p_D is negative, which indicates that the universe is a accelerating of expansion. The energy density ρ_D shown in figures 8 and 12 for both cases is divers as $t \to 0$ and becomes zero as $t \to \infty$. From figures 9 and 13 We observed that for both the cases that during the evolution of the universe the behavior of EoS parameter ω_D is a decreasing function of cosmic time t. From equation 40 and 50 we observed that in non-interacting case the total energy density parameter has the same properties as in interacting case.

8. CONCLUSIONS

In the present paper, we have studied the system of two fluid scenario for dark energy parameter in the spatial homogenous and anisotropic five dimensional Kaluza-Klein space time field with a barotropic fluid and dark energy by considering a variable deceleration parameter. For the model the spatial volume increase as $t \to \infty$. The anisotropic parameter Δ and shear scalar σ^2 tends to zero as cosmic time $t \to \infty$. The pressure p_D is negative, which indicates that the universe is a accelerating of expansion. The energy density ρ_D is divers as $t \to 0$ and becomes zero as $t \to \infty$. The EoS parameter ω_D is a decreasing function of cosmic time t in both the cases and is always varying in the quintessence region. The total energy density parameter Ω_D tends to one at late time. The value of jerk parameter $j = 1$, deceleration parameter $q = -1$ predict that the universe decelerating expansion to accelerating expansion passes through a transition phase. This solution gives the model tends to the ΛCDM model at late time.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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Anisotropic cloud string cosmological model with five-dimensional kaluza-klein space-time

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Anisotropic cloud string cosmological models has been investigated in the context of five dimensional Kaluza- Klein space time. In this paper the energy momentum tensor is generated by rest energy density and tension density of the string with particle density attached to them. To obtained the exact solutions of the Einstein field equations we assumed a scale factor $a(t) = e^{\frac{1}{\beta}\sqrt{2\beta t+c}}$ where β and c are positive constant, which yields a variable deceleration parameter (DP) $q = -\frac{a\ddot{a}}{\dot{a}^2} = \beta H + \alpha$. The physical and geometrical behavior of the models is also discussed in detail.

KEYWORDS

kaluza-klein space time, string tension density, particle density, variable deceleration parameter, cloud string

Introduction

Now a day, string cosmology has attracted lots of attention, because of its significant role in the study of the origin and evaluation of the Universe before the creation of particles. It is a fascinating field for cosmologists to study and discover the mysterious phenomena that have yet to be observed and explore the unseen information of our Universe. As a result, cosmologists are extremely interested in learning more about the past, present, and future evolution of the Universe. But, as of now, we lack strong evidence to make a conclusive statement about its origin and evolution. So, further investigation is required to discover the mysterious phenomena of the entire universe. Stachel (1980) and Letelier (1983) was started the study of string in the context of general relativity. Because the string is extremely appropriate in describing the early phase of the evolution of our Universe. Many eminent authors are interested to work in the field of cosmic strings within the context of general relativity (Kibble, 1976; 1983), and it is thought that strings cause density perturbations that lead to the formation of massive scale structures in the Universe (Zel'dovich et al., 1974; Zel'dovich, 1980).

Strings are stable topological structures that formed during the early universe phase transition due to a drop in temperature below certain critical temperatures. Observations of our universe using contemporary technical tools also suggest that in the early stages of our Universe, there existed a massive scale network of strings. Geometric strings and massive strings are two types of strings that contain stress-energy. The presence of strings is responsible for the universe is anisotropy; nevertheless, strings are no longer visible. These strings are not damaging the cosmological models, instead, they can lead to a variety of fascinating astrophysical results. Strings can also be used to describe the nature and essential arrangement of the early Universe. String theory describes the early stage of evolution of the Universe in terms of (vibrating) strings instead of particles and gives us a single theoretical structure in which all matter and forces are unified. Because strings play such an important role in describing the evolution of the early stages of our Universe. Several authors have recently focused their attention on string cosmological models. According to GUT (grand unified theories), after the big-bang explosion, there is a symmetry flouting during the phase transition of the early stages of the Universe, and these strings appear when the cosmic temperature descends below certain critical temperatures (Everett, 1981; A. Vilenkin, 1981a,b).

The study of Kaluza-Klein (KK) (Kaluza, 1921; Klein, 1926) theory is a model that sought to integrate Einstein's theory of gravity and Maxwell's electromagnetism theory, which revolves around the concept of the fifth dimension, beyond the four dimensions of space and time. The study of KK cosmology became popular because of its illustrious history and some interesting features to revolutionize the study of the universe. This allows the universe to expand early and study its evolution and behavior, adding extra dimensions to Einstein's field equations as seen nowadays. It is becoming very fascinating to study string cosmology in higher-dimensional space-time in the context of general relativity. Several researchers like Chodos and Detweiler (1980), Appelquist et al. (1987) have investigated a homogeneous higher dimensional cosmological model with massive string in general relativity. Naidu et al. (2013) and Reddy and Lakshmi (2014) have explored the possibility of higher dimensional space-time in the field of cosmology. Jain and Shyamsunder (2015), Khadekar and Patki (2008), Sharif and Khanum (2011), Venkateswarlu and Kumar (2006), Khadekar and Vaishali (2010), Samanta and Dhal (2013), Raut et al. (2015) have discussed five-dimensional KK cosmological models with different matters. Adhav et al. (2008) and Yilmaz (2006) have investigated KK cosmic solutions are examined in higher dimensions for quark matter along with string cloud and domain walls in the context of general relativity. Reddy et al. (2007) and Reddy and Naidu (2007) have investigated a higher-dimensional string cosmological model in different theories of gravitation. Khadekar et al. (2008) investigated string dust cosmological models with particles attached to them by considering three different forms of variable Λ in the context of five-dimensional KK space-time. Khadekar et al. (2007) studied a string cosmological model with bulk viscosity in higher dimensional space-time. Nimkar (2017) discussed String cosmological model with the electromagnetic field in general relativity. Pawar et al. (2018) discussed KK string cosmological model in f (R, T) theory of gravity. Krori et al. (1994) have investigated a higher dimensional Bianchi type-I cosmological model with string and they found that matter and string coexist throughout the evolution of the universe. Mohanty et al. (2002), Sahoo et al. (2017) have investigated the anisotropic cosmological model universe in Bianchi type-I space-time. Venkateswarlu and Pavankuma (2005) have investigated a string cosmological model in higher dimensional space-time with scale covariant theory of gravitation. Rahaman et al. (2003) obtained the exact solutions of the field equations for the higher dimensional space time in the framework of Lyra manifold when the source of gravitational field is a massive string. Kandalkar et al. (2012) constructed Bianchi type-III string cosmological models in presence of magnetic field in the context of general relativity and obtained exact solution of the field equations by using the condition that the sum of the energy density and tension density is zero. Mohanty and Samanta (2009) have investigated a five dimensional axially symmetry string cosmological models in general theory of relativity in presence of bulk viscous fluid. Samanta and Debata (2011) constructed Bianchi type-I five dimensional string cosmological model in the framework of Lyra manifold. Choudhury (2017), Tripathi et al. (2021), Dubey et al. (2018), Tiwari et al. (2019), Ram and Verma (2019), Mollah et al. (2019) and Singh and Baro (2020) are some of the eminent authors who studied different string cosmological models in higher dimensional space time in the contexts of the general relativity. Recently Mollah and Singh (2021) and Baro et al. (2021) constructed higher dimensional Bianchi type-III string cosmological in the framework of general relativity.

In this article, we discuss anisotropic cloud string cosmological models with particles attached to them in the five-dimensional KK space-time. This article is prepared as follows: Sec.2 is devoted to the metric and Einstein's field equations. In Sec. 3 we presented the solutions of the field equations. The geometrical and physical interpretation of the results is given in sec. 4. In the last section, we give the conclusions.

Metric and field equations

The five-dimensional KK metric is given by

$$ds^2 = dt^2 - A^2\left(dx^2 + dy^2 + dz^2\right) - B^2 d\phi^2 \qquad (1)$$

where A and B are functions of cosmic time t only and the fifth coordinate ϕ is taken to be extended space like coordinate.

Einstein's field equation is given by

$$R_{ij} - \frac{1}{2} R g_{ij} = -T_{ij} \qquad (2)$$

where R_{ij} is the Ricci tensor R is the Ricci scalar g_{ij} is the metric tensor and T_{ij} is the energy-momentum tensor for a cloud string respectively.

Thus the energy-momentum tensor for a cloud string is given by

$$T_{ij} = \rho v_i v_j - \lambda x_i x^j \tag{3}$$

where v_i and x_i satisfy the conditions

$$v^i v_i = -x^i x_i = -1, v^i x_i = 0 \tag{4}$$

Here ρ is the rest energy density for a cloud of strings with particles attached to them. $\rho = \rho_p + \lambda$, ρ_p being the rest energy density of particles attached to the strings and λ the tension density of the strings. Here p and ρ are a function of cosmic time t only. x_i is a unit space-like vector instead of the direction of strings so that $x^2 = x^3 = x^4 = 0$ and $x^1 \neq 0$.

The energy-momentum tensor T_{ij} in co-moving coordinates for could string is given by

$$T_0^0 = \rho, T_1^1 = \lambda, T_2^2 = T_3^3 = T_4^4 = 0 \tag{5}$$

The field Eq 2 for the line-element (1) with the help of Eqs. 3–5 can be written explicitly as

$$3\frac{\dot{A}^2}{A^2} + 3\frac{\dot{A}\dot{B}}{AB} = \rho \tag{6}$$

$$2\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + 2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}^2}{A^2} = \lambda \tag{7}$$

$$2\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + 2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}^2}{A^2} = 0 \tag{8}$$

$$3\frac{\ddot{A}}{A} + 3\frac{\dot{A}^2}{A^2} = 0 \tag{9}$$

An over dot indicates a derivative with respect to cosmic time t.

The spatial volume for the model (1) is given by

$$V = a^4 = A^3 B \tag{10}$$

The generalized signify Hubble parameter for Kaluza-Klein space time is given by

$$H = \frac{1}{4}\left(\frac{3\dot{A}}{A} + \frac{\dot{B}}{B}\right) \tag{11}$$

The directional Hubble parameters H_x, H_y, H_z and H_ϕ in the direction of x, y, z and ϕ respectively for the Kaluza-Klein metric are

$$H_x = H_y = H_z = \frac{\dot{A}}{A}$$

and

$$H_\phi = \frac{\dot{B}}{B}$$

The scalar expansion θ and shear scalar σ^2 are given by

$$\theta = 4H = \frac{3\dot{A}}{A} + \frac{\dot{B}}{B} \tag{12}$$

$$\sigma^2 = \frac{1}{2}\left[\sum_{i=1}^{4} H_i^2 - 4H^2\right] = \frac{4}{2}\Delta H^2 \tag{13}$$

The expansion of signify anisotropic parameter (Δ) is given by

$$\Delta = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{\Delta H_i}{H}\right)^2 \tag{14}$$

where $\Delta H_i = H_i - H$ and $H_i = 1, 2, 3, 4$ represent the directional Hubble parameters in H_x, H_y, H_z and H_ϕ directions respectively.

Solutions of the field equations

The set of linearly independent field Eqs 6–9 with five unknown A, B, ρ, λ and ρ_p.

To solve the system of equations we consider deceleration parameter (q) as a linear function of hubble parameter (Tiwari et al., 2015; Tiwari et al., 2018; Sharma et al., 2019):

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = \beta H + \alpha \tag{15}$$

Here α and β arbitrary constants.

For $\alpha = -1$ in Eq 15

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -1 + \beta H$$

which yields the following differential equation

$$\frac{a\ddot{a}}{\dot{a}^2} + \beta\frac{\dot{a}}{a} - 1 = 0 \tag{16}$$

After integration Eq 16 we get

$$a(t) = e^{\frac{1}{\beta}\sqrt{2\beta t + c}} \tag{17}$$

where c is an integrating constant.

Collins et al. (1980) have exposed that for a spatially homogeneous metric, a large class of solutions that can satisfy the condition $\frac{\sigma}{\theta}$ is constant, where θ is the expansion in the model. So we assume the shear scalar σ is proportional to the expansion scalar θ. This gives the relation between scale factor A and B as,

$$A = B^n \tag{18}$$

where n is constant and $n \neq 1$.

From Eqs. 10, 17, 18 the metric component are

$$A(t) = e^{\frac{4n}{\beta(3n+1)}\sqrt{2\beta t + c}} \tag{19}$$

and

$$B(t) = e^{\frac{4}{\beta(3n+1)}\sqrt{2\beta t + c}} \tag{20}$$

Therefore the metric (1) reduce to

$$ds^2 = dt^2 - e^{\frac{4n}{\beta(3n+1)}\sqrt{2\beta t + c}}\left(dx^2 + dy^2 + dz^2\right) - B(t) - e^{\frac{4}{\beta(3n+1)}\sqrt{2\beta t + c}} d\phi^2 \tag{21}$$

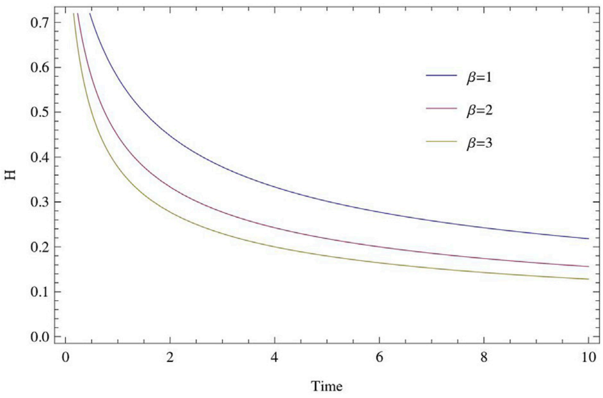

FIGURE 1
The plot of Hubble parameter H versus time t.

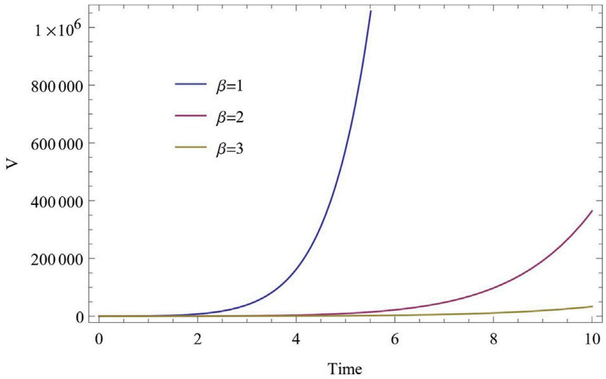

FIGURE 2
The plot of spatial volume *V* versus time t.

Eq. 21 represents Five-Dimensional KK Cosmological Models with variable deceleration parameter.

Physical properties of the model

We have obtained the cosmological model (21), the directional Hubble parameters H_x, H_y, H_z and H_ϕ, the physical quantities such as Hubble parameter H, spatial volume V, signify anisotropy parameter Δ, expansion scalar θ, shear scalar σ^2, energy density ρ, particles density ρ_p and tension density of the string λ are obtained as follows:

The directional Hubble parameters H_x, H_y, H_z and H_ϕ are

$$H_x = H_y = H_z = \frac{4n}{(3n+1)\sqrt{2\beta t + c}}$$

and

$$H_\phi = \frac{4}{(3n+1)\sqrt{2\beta t + c}}$$

For Kaluza-Klein space-time, the signify Hubble parameter(H) is given by

$$H = \frac{1}{\sqrt{2\beta t + c}} \tag{22}$$

The spatial volume(V) is given by

$$V = e^{\frac{4}{\beta}\sqrt{2\beta t + c}} \tag{23}$$

The expansion of signify anisotropic parameter (Δ) is given by

$$\therefore \Delta = \frac{3(n-1)^2}{(3n+1)^2} = constant\,(\neq 0 \; where \; n \neq 1) \tag{24}$$

The expansion scalar (θ) is given by

$$\theta = \frac{4}{\sqrt{2\beta t + c}} \tag{25}$$

The shear scalar (σ^2) is given by

$$\sigma^2 = \frac{3(n-1)^2}{2(3n+1)^2(2\beta t + c)^2} \tag{26}$$

From Eqs 25, 26 we obtain

$$\lim_{t\to\infty} \frac{\sigma^2}{\theta^2} = \frac{3(n-1)^2}{8(3n+1)^2} = constant\,(\neq 0 \; where \; n \neq 1) \tag{27}$$

The energy density ρ is given by

$$\rho = \frac{48n(n+1)}{(3n+1)^2(2\beta t + c)} \tag{28}$$

The tension density λ for the string is given by

$$\lambda = \frac{16(3n^2+2n+1)}{(3n+1)^2(2\beta t + c)} - \frac{4\beta(2n+1)}{(3n+1)(2\beta t + c)^{\frac{3}{2}}} \tag{29}$$

The particles density ρ_p is obtained by

$$\rho_p = \frac{16(n-1)}{(3n+1)^2(2\beta t + c)} + \frac{4\beta(2n+1)}{(3n+1)(2\beta t + c)^{\frac{3}{2}}} \tag{30}$$

The deceleration parameter (q) is given by

$$q = -1 + \frac{\beta}{\sqrt{2\beta t + c}} \tag{31}$$

It can be seen that from Eqs. 22, 25, both the Hubble parameter (H) and expansion scalar (θ) is a positive and decreasing function of cosmic time t. The Hubble parameter (H) and expansion scalar (θ) tend to infinity as $t \to 0$ and tend to a finite value as $t \to \infty$ are shown in Figures 1, 3 which are agrees with established theories. Figure 2

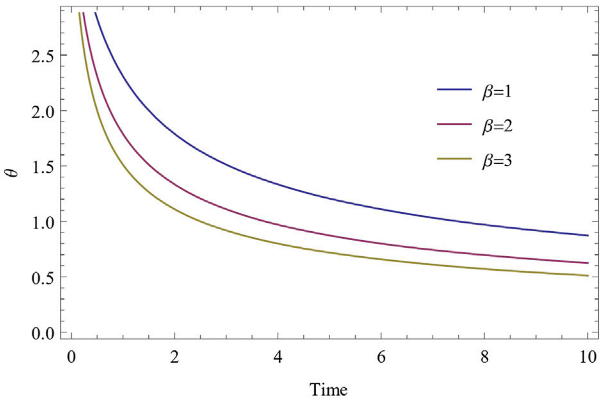

FIGURE 3
The plot of expansion scalar θ versus time t.

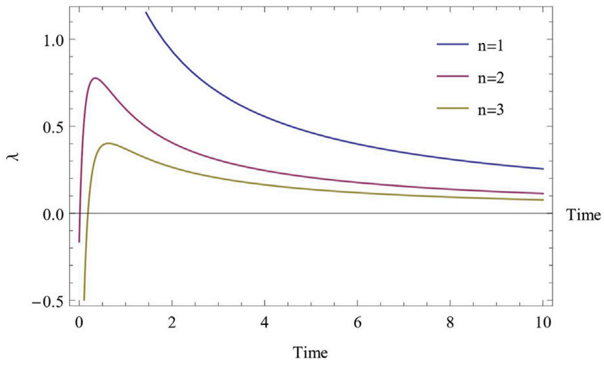

FIGURE 5
The plot of tension density of the string λ versus time t.

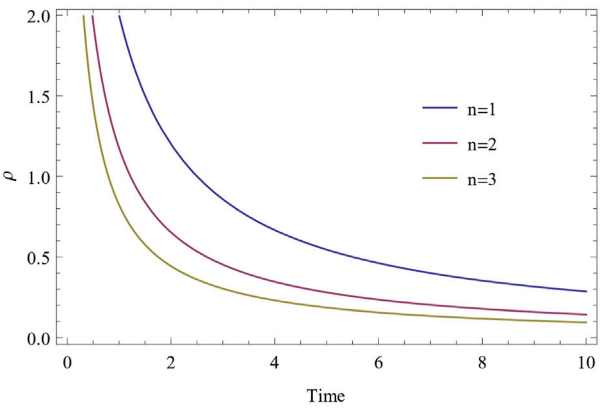

FIGURE 4
The plot of energy density ρ versus time t.

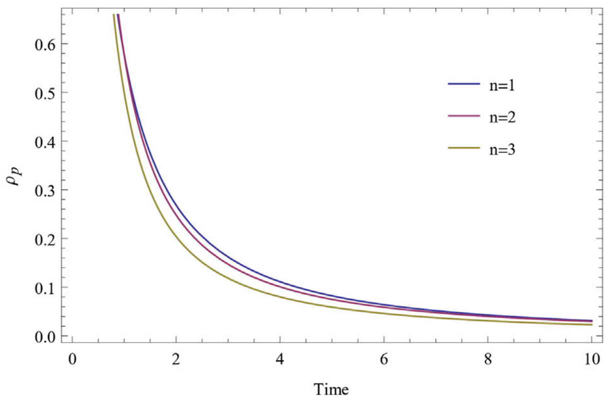

FIGURE 6
The plot of the particles density ρ_p versus time t.

shows variation of spatial volume w.r.t. time. We have also noticed that $\frac{dH}{dt}$ is negative which indicates that our universe is expanding rapidly.

From Eq. 23 shows that at $t = 0$ the spatial volume is finite and thereafter increases continuously when cosmic time t is increasing. Figure 3 depicts the nature of variations of V versus t.

From Eq. 31 it is observed that the deceleration parameter $q > 0$ for $t < \frac{\beta^2 - c}{2\beta}$ which indicates that our model universe is a decelerating phase. It is also observed that the deceleration parameter $q < 0$ for $t > \frac{\beta^2 - c}{2\beta}$ which indicates that our model universe is a accelerating phase, which agrees with present day's observations (Riess et al., 1998; Perlmutter et al., 1999).

The expansion of signify anisotropic parameter $\Delta \neq 0$ (constant) for $n \neq 1$ and $\Delta = 0$ for $n = 1$. We also observed from Eq. 27 that $\lim_{t\to\infty} \frac{\sigma^2}{\theta^2} \neq 0$ (constant) for $n \neq 1$ and $\lim_{t\to\infty} \frac{\sigma^2}{\theta^2} = 0$ for $n = 1$, which means that our model is anisotropic when $n \neq 1$ and it is isotropic when $n = 1$.

From Eq. 28 it is seen that the expansion for rest energy density ρ is a decreasing function of cosmic time t. This shows that the rest energy density is positive and satisfies the condition of energy $\rho \geq 0$ for all $n \geq -1$. Also from Figure 4, it is seen that the rest energy density ρ is decreasing when time t is increasing and initially $\rho \to \infty$ when $t \to 0$, thus has an initial singularity.

It is seen from Figures 5, 6 that both the string tension density λ and particle density ρ_p are positive, decreasing function of cosmic time t, and become zero as $t \to \infty$. Also, we observed that initially both the string tension density λ and particle density ρ_p tend to infinity when t tends to zero which suggests that the universe began with big bang and as time progresses, both the string tension density λ and particle density ρ_p decreases with the expansion of the universe.

Conclusion

In the present article, we have investigated the behavior of anisotropic cloud string cosmological models in five-dimensional KK space-time to describe the mysterious phenomena of the entire universe. To get the exact solutions of the Einstein field equations, we assumed a scale factor $a(t) = e^{\frac{1}{\beta}\sqrt{2\beta t+c}}$ where β and c are positive constant, which yields a variable deceleration parameter (DP) $q = -\frac{a\ddot{a}}{\dot{a}^2} = \beta H + \alpha$. Our model depicts to have an anisotropic phase for $n \neq 1$ throughout the evolution of the universe as it does not depend on the cosmic time t. According to present day's observations, there is a disparity in measuring microwave intensity from different directions of the sky. This motivated us to investigate the universe using the anisotropic five dimensional Kaluza-Klein space-time in order to better describe our universe. Several cosmological observations such as Cosmic Background Explorer (COBE) and the Wilkinson Microwave Anisotropic Probe (WMAP) are also evidence that we live in a globally anisotropic universe. In order to produce any significant amount of shear in recent periods, one must cause anisotropy in space-time and WMAP, where they found small anisotropy in microwave background radiation. Also, the models represents an exponentially expanding Universe that begins with the big bang at cosmic time t = 0 with finite volume and extends at an accelerating rate. The deceleration parameter "q" of the universe has certainly changed its sign from positive to negative (signature flipping), which indicates that the universe has decelerated expansion in the past and accelerated expansion at present day's observations (Amendola, 2003; Padmanabhan and Choudhury, 2003; Kandalkar and Samdurkar, 2015). Our model satisfies the condition of energy density $\rho \geq 0$ and $\rho_p \geq 0$. The particle density and string tension density are equivalent, but the string tension density vanishes faster than the particle density, so our model reflects a matter-dominated universe that accords with current observational data in the late time period.

Data availability statement

The original contributions presented in the study are included in the article, further inquiries can be directed to the corresponding author.

Author contributions

All of the authors listed have contributed a significant, direct, and intellectual contribution to the work and have given their permission for it to be published.

Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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