A STUDY ON COSMOLOGICAL MODELS IN FIVE DIMENSIONAL KALUZA-KLEIN SPACE-TIME

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Chapter 8

Conclusion and Scope for Future Research

In this thesis, we have investigated the cosmological models of the universe in five dimensional Kaluza-Klein space-time within the framework of time dependent deceleration parameter and dynamical cosmological term Λ. The vital role of this investigation is the construction of the cosmological models with specific theoretical framework, which have been covered in the chapters of the thesis. The construction of the models is primarily based on a average scale factor and a cosmological term as Λ approach, where the main goal is to constrain the values of the model parameters from some physically plausible basis and then to determine the evolution of various cosmological quantities, such as the spatial volume, Hubble parameter, anisotropic parameter, deceleration parameter, EoS parameter, statefinder parameters, etc.

In chapter-2, we deal with five dimensional Kaluza-Klein dark energy cosmological models of the universe under two fluid scenario within the framework of variable deceleration parameter. The constructed models behave like a quintessence region with $-1 < \omega_D < -\frac{1}{3}$. The total energy density parameter Ω tends to one as $t \to \infty$. The value of jerk parameter $j = 1$ and the deceleration parameter $q = -1$ predicts that the model universe tends to the ΛCDM model ($r = 1, s = 0$) at late time of cosmic evolution. The proposed model satisfied the weak energy condition (WEC), null energy condition (NEC) and dominant energy condition (DEC) but violated the strong energy condition (SEC). Thus we can say that the derived model agrees with the established theoretical results and present day's observations.

In chapter-3, we have investigated the behavior of anisotropic cloud string cosmological models in five-dimensional Kaluza-Klein space-time to describe the enigmatic phenomena of the entire universe. The derived model behave like anisotropic phase for $n \neq 1$ and it reaches the phase of isotropy when $n = 1$ throughout the evolution of the universe as it does not depend on the cosmic time t. The model represents an exponentially expanding universe that begins with the big bang at cosmic time $t = 0$ with finite volume and extends at an accelerating rate. The proposed model satisfies the condition of energy density $\rho \geq 0$ and $\rho_p \geq 0$. The particle density and string tension density are equivalent, but the string tension density vanishes faster than the particle density, so the model reflects a matter-dominated universe that accords with current observational data in the late time cosmic evolution.

In chapter-4, we have investigated the role of homogeneous and isotropic five dimensional Kaluza-Klein space-time for all three types of open, closed and flat universe. In this chapter, we have investigated the physical and geometrical importance of the two fluid scenario is described in various aspects. The proposed model work for all three open, closed and flat universe. All the physical parameters like ρ_D, p_D and ω_D for both non-interacting and interacting cases are consistent with established theoretical results and recent observations. We also discussed the jerk parameter in our derived models.

In chapter 5, we have investigated the five dimensional Kaluza-Klein cosmological models with time dependent gravitational and cosmological constant in conharmonically flat space. The scale factor, spatial volume and gravitational constant G becomes infinity as $t \to \infty$. The energy density and pressure diverges as $t \to 0$ and becomes zero as $t \to \infty$. The cosmological constant Λ is infinite as $t \to 0$ and as $t \to \infty$ cosmological constant Λ converging to a small positive value. The nature of decaying vacuum energy density Λ in our derived models is supported by recent cosmological observations ((Perlmutter et al. (1998)), (Riess et al. (1998))). We observed that our derived models are isotropic at present epoch which is in good agreement with the recent observations.

Chapter 6, we deal with a spatially homogeneous and isotropic FRW type five dimensional Kaluza-Klein cosmological models with dynamical cosmological term Λ in conharmonically flat space in presence of perfect fluid. Here we have got the model universe represent an expanding with an accelerated rate. The scale factor and spatial volume becomes infinity as $t \to \infty$ whereas the Hubbles parameter H and θ energy density ρ, pressure p and cosmological constant Λ converge to zero as $t \to 0$.

From the observation it is found that the cosmological constant Λ to be a small positive value at the late time of evolution which is supported by present day's observational data (Riess et al. (1998); (Perlmutter et al. (1999)). Initially when $t \to 0$, the energy density $\rho \to \infty$, which has an initial singularity. The statefinder parameter $(r = 1, s = 0)$ approached to ΛCDM model, which agreed with prevailing theories.

Chapter 7, deals with FRW type five dimensional Kaluza-Klein cosmological model in conharmonically flat space with the help of hybrid expansion law for the average scale factor. The derived model universe represent an expanding, shearing and non-rotating universe. The scale factor and spatial volume becomes infinity as $t \to \infty$. The Hubble parameter, expansion scalar, energy density, pressure all are decreasing function of cosmic time t. The cosmological constant Λ is infinite as $t \to 0$ and tends to small positive value as $t \to \infty$. The behaviour of the statefinder parameter $(r, s) \to (1, 0)$ at late time evolution, which supports the present observational data.

In this work , we have found that:

- The proposed models of the universe, it is observed that the models are accelerating expansion during the evolution of the universe.
- The consideration of the time-dependent deceleration parameter describes the scenarios that cause the universe has an accelerated phase of expansion and the value of deceleration parameter (q) can be found nearby in the range $-1 < q < 0$ which is in good agreement with current observations of Type Ia supernovae ((Riess et al. (1998)); (Garnavich et al. (1998)); (Schmidt et al. (1998)); (Perlmutter et al. (1999))).
- The results obtained for most of the models show that the models are in close proximity of ΛCDM model during the evolution of the universe.

- In derived models of the universe, most of the cosmological parameter H, θ, σ^2 and Δ diverge and there is a point type singularity (MacCallum (1971)).

- The proposed model satisfies the WEC, NEC and DEC but violated the SEC. Thus we can say that the derived model agrees with the established theoretical results and present day's observations.

In the present thesis, the proposed models are in good agreement with the present day's observational data. The study of 5D Kaluza-Klein space-time has been established as an appealing and broad area of research with numerous applications in the field of cosmology. The present study reports good agreement between derived and experimental data. Further study of this type of investigation provides us with the idea that our present universe is much greater than the universe at the early stage of evolution due to the accelerated expansion of the universe. So, nowadays it is becoming very fascinating to study the universe in 5D Kaluza-Klein space-time in the framework of general relativity.

These findings have encouraged new researchers to study the 5D Kaluza-Klein cosmology. As a result, we have a broad scope to examine new research problems in this domain. In future, the 5D Kaluza-Klein space-time can be suitably extended to study the physical and geometric behaviour of the cosmological model of the universe in the different theories of gravitation.