

A STUDY ON COSMOLOGICAL MODELS IN FIVE DIMENSIONAL KALUZA-KLEIN SPACE-TIME

*A Thesis Submitted to Bodoland University, Kokrajhar
in Partial Fulfillment for the Award of the Degree of*
DOCTOR OF PHILOSOPHY IN MATHEMATICS

By

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Ph. D. REGISTRATION NO. : FINAL/MAT00287 of 2019-2020

Under the Supervision of
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2023



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CERTIFICATE

This is to certify that the thesis entitled "A Study on Cosmological Models in Five Dimensional Kaluza-Klein Space-Time" being submitted by Pranjal Kumar Ray for the award of the Degree of Doctor of Philosophy in Mathematics to Bodoland University, Kokrajhar, Assam, India is a record of bonafide research work carried out by him under my supervision and guidance in the Department of Mathematical Sciences, Bodoland University, Kokrajhar, Assam, India.

The thesis satisfies the requirements of the regulations relating to the degree. A considerable part of the thesis is in process to publish in International Journals and some of it has been presented in seminars. Any part of this thesis has not been submitted to other Universities/Institution for awarding any degree or diploma.

Dated, Kokrajhar
The 30th May, 2023

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Declaration

I, **Pranjal Kumar Ray**, hereby declare that the work presented in the thesis entitled, “**A Study on Cosmological Models in Five Dimensional Kaluza-Klein Space-Time**”, is the record of original work done by me under the guidance and supervision of **Dr. Rajshekhar Roy Baruah**, Assistant Professor, Department of Mathematical Sciences, Bodoland University, Kokrajhar. This thesis is being submitted to the Bodoland University, Kokrajhar-783370, Assam, India, in partial fulfillment for the award of the degree of **Doctor of Philosophy in Mathematics**.

I further declare that, this research work described in the thesis is original and has not been submitted previously to any other University or Institution for the award of any other degree or diploma.

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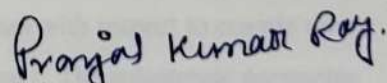
Acknowledgement

It gives me immense pleasure to express my deep sense of gratitude to my supervisor **Dr. Rajshekhar Roy Baruah**, Assistant professor, Department of Mathematical Sciences, Bodoland university, Kokrajhar, Assam, India for his all time support, inspiring guidance, constant encouragement, meticulous supervision, stimulating discussion and valuable suggestions during my research work and preparation of this thesis. I am highly indebted to him for the help.

I express my sincere thanks to **Dr. Bhimraj Basumatary**, HoD, Department of Mathematical Sciences, Bodoland University and all the Teaching staff as well as Non-Teaching staff and Research Scholars of the department of Mathematical Sciences, Bodoland university for their useful suggestions, constant encouragement and helpful advice during the progress of my research work.

I would like to express my gratitude to all the faculty members of the Department of Mathematics and Statistics, Kokrajhar Govt. College, Kokrajhar, Assam for their useful suggestions and helpful advice during my research period.

Last but not the least; I shall always remain indebted to all my family members and friends for their unconditional love, blessings, support and inspiration throughout the research work.



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List of Figures

2.1	(a) The plot of V versus cosmic time t , (b) The plot of deceleration parameter q versus cosmic time t , for $\beta = 1$ and $c = 1$	38
2.2	(a) The plot of H versus cosmic time t , (b) The plot of expansion scalar θ versus cosmic time t , for $\beta = 1$ and $c = 1$	38
2.3	(a) The plot of Δ versus cosmic time t , (b) The plot of shear scalar σ^2 versus cosmic time t , for $\beta = 1$, $\lambda = 1$ and $c = 1$	39
2.4	(a) The plot of p_D versus cosmic time t for non- interactive case, (b) The plot of pressure p_D versus cosmic time t for interactive case, for $\beta = 1, c = 1, \lambda = 1, \rho_0 = 1$ and $\omega_m = 0.5$	39
2.5	(a) The plot of ρ_D versus cosmic time t for non-interactive case, (b) The plot of energy density ρ_D versus cosmic time t , for interactive case, for $\beta = 1, c = 1, \lambda = 1, \rho_0 = 1, k = .02$ and $\omega_m = 0.5$	40
2.6	(a) The plot of ω_D versus cosmic time t for non-interactive case, (b) The plot of ω_D versus cosmic time t for interactive case, for $\beta = 1, c = 1, \lambda = 1, \rho_0 = 1, k = .02$ and $\omega_m = 0.5$	40
2.7	(a) The plot of energy condition versus cosmic time t for non-interactive case, (b) The plot of energy condition versus cosmic time t for interactive case, for $\beta = 1, c = 1, \lambda = 1, \rho_0 = 1, k = .02$ and $\omega_m = 0.5$	41
3.1	The plot of H versus cosmic time t	48
3.2	The plot of θ versus cosmic time t	49
3.3	The plot of V versus cosmic time t	49
3.4	The plot of ρ versus cosmic time t	50
3.5	The plot of λ versus cosmic time t	50
3.6	The plot of the ρ_p versus cosmic time t	51
4.1	The plot of ρ_D versus cosmic time t with $m = 1, \rho_0 = 1, \alpha = 3, \omega_m = 0.5$ and $k = -1, 0, +1$ for non-interactive case.	58

4.2	The plot of ρ_D versus cosmic time t with $m = 1, \rho_0 = 1, \alpha = 3, \omega_m = 0.5$ and $k = -1, 0, +1$ for interactive case.	59
4.3	The plot of p_D versus cosmic time t with $m = 1, \rho_0 = 1, \alpha = 3, \omega_m = 0.5$ and $k = -1, 0, +1$ for non-interactive case.	59
4.4	The plot of p_D versus cosmic time t with $m = 1, \rho_0 = 1, \alpha = 3, \omega_m = 0.5$ and $k = -1, 0, +1$ for interactive case.	59
4.5	The plot of ω_D versus cosmic time t with $m = 1, \rho_0 = 1, \alpha = 3, \omega_m = 0.5$ and $k = -1, 0, +1$ for non-interactive case.	60
4.6	The plot of ω_D versus cosmic time t with $m = 1, \rho_0 = 1, \alpha = 3, \omega_m = 0.5$ and $k = -1, 0, +1$ for interactive case.	60
4.7	The plot of q versus cosmic time t with $m = 1, \alpha = 3, 4, 5$	60
4.8	The plot of j versus cosmic time t with $m = 1, \alpha = 3, 4, 5$	61
5.1	The plot of q versus cosmic time t , for $\beta = 1, 1.5, 2, 2.5, c = 1$	67
5.2	The plot of G versus cosmic time t , for $\beta = 1, m = 0.5, 1, 1.5$	71
5.3	The plot of Λ versus cosmic time t , for $\beta = 1, \omega = 0, 1, \frac{1}{3}$	72
5.4	The plot of ρ versus cosmic time t , for $\beta = m = 1, \omega = 0, 1, \frac{1}{3}$	72
5.5	The plot of p versus cosmic time t , for $\beta = m = 1, \omega = 0, 1, \frac{1}{3}$	73
6.1	The plot of $a, V, H, \theta, \rho, \Lambda$ versus cosmic time t , for $\omega = 0, \alpha_1 = \alpha_2 = 1$	80
6.2	The plot of $a, V, H, \theta, \rho, \Lambda$ versus cosmic time t , for $\omega = 1, \alpha_1 = \alpha_2 = 1$	81
6.3	The plot of $a, V, H, \theta, \rho, \Lambda$ versus cosmic time t , for $\omega = \frac{1}{3}, \alpha_1 = \alpha_2 = 1$	82
7.1	The plot of H and θ versus cosmic time t , for $\alpha_1 = \alpha_2 = 1$	90
7.2	The plot of q versus cosmic time t	90
7.3	The plot of ρ versus cosmic time t , for $a_0 = \alpha_1 = \alpha_2 = 1, \omega = 0, 1, \frac{1}{3}$	91
7.4	The plot of P versus cosmic time t , for $a_0 = \alpha_1 = \alpha_2 = 1, \omega = 0, 1, \frac{1}{3}$	91
7.5	The plot of Λ versus cosmic time t , for $a_0 = \alpha_1 = \alpha_2 = 1, \omega = 0, 1, \frac{1}{3}$	92

List of Abbreviations

KK : Kaluza-Klein
5D : Five-Dimensional
GR : General Relativity
GTR : General Theory of Relativity
FLRW: Friedmann Lemaitre Robertson Walker
FRW : Friedmann Robertson Walker
DE : Dark Energy
CMB : Cosmic Microwave Background
COBE : Cosmic Background Explorer
WMAP : Wilkinson Microwave Anisotropy Probe
HDM : Hot Dark Matter
WDM : Warm Dark Matter
CDM : Cold Dark Matter
SMC : Standard Model of Cosmology
SMPP : Standard Model of Particle Physics
EoS : Equation of State
EMT : Energy Momentum Tensor
SN Ia : Type Ia Supernova
SEC : Strong Energy Condition
WEC : Weak Energy Condition
NEC : Null Energy Condition
DEC : Dominant Energy Condition

SCDM : Standard Cold Dark Model

CMBR : Cosmic Microwave Background Radiation

SDSS : Sloan Digital Sky Survey

VDP : Variable Deceleration Parameter

DP : Deceleration Parameter

List of Symbols

a : Scale factor

H : Hubble's parameter

G : Gravitational constant

Λ : Cosmological Constant

V : Spatial Volume

P : Pressure

Δ : Anisotropic parameter

g_{ij} : Metric tensor

ρ : Energy density

ω : Equation of state (EoS)

σ^2 : Shear scalar

q : Deceleration parameter

R_{ij} : Ricci tensor

R : Ricci scalar

T_{ij} : Energy momentum tensor

θ : Expansion scalar