Abstract

This thesis entitled **"A Study on Cosmological Models in Five Dimensional Kaluza-Klein Space-Time"** has been devoted to investigations of the nature of the physical universe. This thesis consists of 8 (eight) chapters and deals with Five Dimensional Kaluza-Klein cosmological models.

Chapter 1, deals with the introductory nature of this research work. In this chapter, we have highlighted some definitions and basic facts related to the foundation and development of cosmology, some important cosmological parameters, strings and string cosmology, Kaluza-Klein theory, some observational evidence of dark energy, the present accelerated expansion of the universe and a review of some work by other authors related to the topic. Further, the objectives, methodology and tools of the research work are also presented.

Chapter 2, deals with the evolution of the dark energy parameter in the spatial homogeneous and anisotropic 5D Kaluza-Klein space-time filled with barotropic fluid and dark energy. To obtain the exact solutions of the Einstein field equations by considering the variable deceleration parameter. Here we consider two cases; first, when these fluids are assumed to be not interacts with each other and second, when they interact with each other. In this chapter, we observed that the behaviour of spatial volume V is zero at initial epoch $t = 0$ and it increases with respect to cosmic time t which shows that the universe is expanding with time t. The anisotropic parameter Δ and shear scalar σ^2 tends to zero as cosmic time $t \to \infty$. We observed that $q < 0$, i.e., at present time we obtain the accelerating phase of the universe. When $t \to \infty$, the models asymptotically approach the value $q = -1$, corresponding to the ΛCDM

model. The pressure p_D is negative and energy density ρ_D is positive, which shows that the model represents a dark energy universe, as established by current observations (Perlmutter et al. (1999); Riess et al. (2004)). The EoS parameter ω_D is a decreasing function of cosmic time t in both the cases and is varying in the quintessence region $(-1 < \omega_D < -\frac{1}{3})$. The total energy density parameter Ω tends to one, when $t \to \infty$. The value of deceleration parameter $q = -1$ predicts that the model universe tends to the ΛCDM model ($r = 1, s = 0$) throughout the evolution of the universe. The proposed model satisfies WEC, NEC and DEC but violated SEC.

Chapter 3, deals with anisotropic cloud string cosmological model in the framework of 5D Kaluza-Klein space-time. In this chapter, the energy momentum tensor is generated by rest energy density and tension density of the string with particle density attached to them. To obtain the exact solutions of the Einstein field equations we assumed a scale factor $a(t) = e^{\frac{1}{\beta}\sqrt{2\beta t+c}}$. Here the derived model depicts an anisotropic phase for $n \neq 1$ and it reaches the phase of isotropy when $n = 1$ throughout the evolution of the universe and it does not depend on the cosmic time t. Also, the model represents an exponentially expanding universe that begins with the Big Bang at cosmic time $t = 0$ with a finite volume and extends at an accelerating rate. The proposed model satisfies the condition of energy density $\rho \geq 0$ and $\rho_p \geq 0$. The particle density and string tension density are equivalent, but the string tension density vanishes faster than the particle density, so the derived model reflects a matter-dominated universe that accords with current observational data in the late time period.

Chapter 4, deals with homogeneous and isotropic 5D Kaluza-Klein dark energy cosmological model filled with barotropic fluid and dark fluid. In this chapter we considered the special form of deceleration parameter $q = \frac{-a\ddot{a}}{\dot{a}^2} = -1 + \frac{\alpha}{1+a^{\alpha}}$ to obtain the exact solution of the Einstein field equations. Here we observed that the derived model worked for all three values of open (for $k = -1$), closed (for $k = 1$) and flat (for $k = 0$) universe. All the physical parameters like ρ_D, p_D and ω_D for both non-interacting and interacting cases are consistent with established theoretical results and recent observations. We also observed that, when $t \to \infty$, Ω approaches to one for all the three values of k (for $k = -1, 0, 1$), which shows that the universe will acquire a flat structure. Here the deceleration parameter q is negative and jerk parameter j is positive so that we do have a transition of the model from decelerated to an accelerated phase. These results are fully consistent with the present day's observations.

Chapter 5, deals with conharmonically flat space with variable gravitational and cosmological constant for a spatial homogeneous and isotropic 5D Kaluza-Klein cosmological model filled in the presence of perfect fluid. In this chapter, we studied a cosmological model with variable deceleration parameter (DP) $q = -\frac{a\ddot{a}}{\dot{a}^2} = \beta H + \alpha$ which gives the scale factor as $a(t) = e^{\frac{1}{\beta}\sqrt{2\beta t + c}}$. Here the scale factor a, spatial volume V and gravitational constant G becomes infinity as $t \to \infty$. The energy density and pressure diverges as $t \to 0$ and becomes zero as $t \to \infty$. The cosmological constant Λ is infinite as $t \to 0$ and as $t \to \infty$ the cosmological constant Λ converges to a small positive value. The nature of decaying vacuum energy density Λ in the models is supported by recent cosmological observations (Perlmutter et al. (1998); Riess et al. (1998)). Here the derived model depicts an isotropic at the present epoch which is in good agreement with the recent observations.

Chapter 6, deals with a spatially homogeneous and isotropic FRW type 5D Kaluza-Klein cosmological model with dynamical cosmological term Λ in conharmonically flat space in the presence of perfect fluid. Here we have the model universe which is expanding at an accelerated rate. The scale factor and spatial volume becomes infinity as $t \to \infty$ whereas the Hubble's parameter H and expansion scalar θ energy density ρ, pressure p and cosmological constant Λ converges to zero as $t \to 0$. We have found the cosmological constant Λ to have a small positive value as $t \to \infty$, which is supported by present day's observational data (Riess et al. (1998); Perlmutter et al. (1999)). Initially when $t \to 0$ the energy density $\rho \to \infty$, which gives rise to an initial singularity. The statefinder parameter $(r = 1, s = 0)$ approaches to ΛCDM model, which agrees with prevailing theories.

Chapter 7, deals with FRW type 5D Kaluza-Klein cosmological model in conharmonically flat space with the help of hybrid expansion law for the average scale factor. The proposed model universe represents an expanding, shearing and non-rotating universe. The scale factor and spatial volume become infinity as $t \to \infty$. The Hubble's parameter, expansion scalar, energy density, pressure are all decreasing function of cosmic time t. The cosmological constant Λ is infinite as $t \to 0$ and tends to small positive value as $t \rightarrow \infty$. The behaviour of the statefinder parameter $(r, s) \rightarrow (1, 0)$ throughout the evolution of the universe, supports the present observational data.

In chapter 8, we summarized the results obtained in each chapter and presented the future scope related to our research work in this thesis.