Chapter 1

Introduction

1.1 The Basic Facts of Cosmology

Cosmology is a discipline of science that studies the whole universe. The name "cosmology" derives from the Greek word "Kosmos," which means "something disclosed in beauty". The study of cosmology includes the determination of large-scale structures of the physical universe as a whole in order to comprehend its origin, development and ultimate destiny. Cosmology is concerned with the numerous theories of the universe creation and develops hypotheses for specific predictions of various phenomena for observations about the universe. The prevailing theory about the origin and evolution of our universe is the so-called Big Bang theory.

The "Big Bang Theory" is now the most commonly accepted theory explaining the origin and ultimate destiny of the physical universe. The fundamental concept of this theory is that the universe is expanding, which implies that there was a very dense and hot universe in the distant past. Cosmology also deals with the issues of understanding the creation of galaxies and clusters, as well as the nature of their masses.

Despite the fact that the universe likely 100 billion galaxies containing billions of stars, dust and gas clouds, perhaps scads of planets, moons, and other small bits of cosmic debris. From radio waves to X-rays, the stars produce a huge amount of energy, which streak across the universe at the speed of light.

1.2 Standard Cosmology

The study of cosmology involves identifying the physical structures of the universe at large scales. It seeks to use scientific methods to understand the origin, evolution and ultimate destiny of the entire universe. The universe is generally defined as the totality of all space, time, matter, energy, planets, stars, galaxies and everything that exists. It contains star clusters, nebulae, pulsars, quarries as well as cosmic rays and background radiation. By focusing on large-scale features, cosmologists create mathematical models which accurately represent the universe, and they compare these models to the cosmos or universe as observed by astronomers. A cosmic model is a representation of our universe that considers and applies all known physical laws, describing in full (nearly) observed properties of the universe and especially the phenomena of the early universe. This hypothesis describes why during the most recent stage of the cosmic microwave of the circumstantial expansion the universe was so uniform and isotropic.

The cosmological expansion is a striking new characteristic that does not exist on small scales, when the universe is viewed in the large. On cosmic scales, galaxies (or, at least, clusters of galaxies) seem to be accelerating away from one another, with the apparent recessional velocity being directly proportional to the distance of object. The Hubble's law is the name of this relationship (after its discoverer, the American astronomer Edwin Powell Hubble's). The Hubble's law suggests that 13.8 billion years ago, when the universe was extremely densely packed, everything suddenly burst into existence in a "Big Bang", with the effects of the explosion eventually being recorded in the galaxies of stars that exist today. The discovery of a constant and uniform microwave background radiation by radio telescopes provides compelling evidence for this theory of the big bang genesis of the universe. The bright brilliance of the primordial fireball, which was diminished by cosmic expansion to a shadow of its former grandeur but is still permeating every part of the physical universe, is thought to be preserved as the cosmic microwave background.

The explanation of the Hubble's law that galaxies recede across over time through space, however, contains a misleading notion. In a way, and this will be clarified later in the text, the expansion of the universe is more like an expansion of time and space than it is a basic motion of galaxies within a framework of absolute space and time. When measuring distances on cosmological scales, light-travel times take on a special significance because the lengths are so large that even light, which travels at the fastest speed possible for any physical object, needs a significant portion of the age of the universe (13.8 billion years old) to travel from one object to another. As a result, when astronomers measure object at cosmic distances from the Local Group, they were viewing the objects as they were when the universe was far more primitive than it is nowadays. In light of this, Albert Einstein argued in his theory of general relativity that the gravitational field of everything in the universe warps space and time to such an extent that it necessitates a very careful re-evaluation of quantities whose ostensibly basic natures are typically taken for granted.

1.3 Kaluza-Klein Theory

The Kaluza-Klein theory, which was based on the idea of a fifth dimension rather than the usual four dimensions of space and time, attempted to bring together Maxwell's theory of electromagnetism and Einstein's theory of gravitation. This theory's fundamental premise was to post an extra short space dimension and introduce nothing but pure gravity in the new (1 + 4)-dimensional space-time. It turns out that our observable (1 + 3)-dimensional space-time is exposed as a gravitational, electromagnetic and scalar field in the fifth dimension of gravity.

The original hypothesis of the five-dimensional theory came from the German mathematician and physicist Theodore Kaluza, who extended general relativity to a new theory in five-dimensional space, and the theory was first published in 1921. The resulting equations can be further divided into several other equations, one of which is equivalent to the Einstein field equation, another set equal to Maxwell's equation for the electromagnetic field and the final part is now referred to as the "radion".

In 1926, Oskar Klein suggested that the fourth spatial dimension as a circle with a very small radius such that a particle travelling down that axis just a short distance would end up back where it started. The size of the dimension is the distance a particle may travel before returning to its starting point. Compactification is the term used to describe the phenomena of having a space-time with compact dimensions. Electromagnetism may be seen as gauge theory on a fibre bundle, the circle bundle, with the circle group U(L) representing the additional fifth dimension in contemporary geometry U(L). Once this geometrical meaning is grasped, a generic Lie group may easily take the place of U(L). Yang-Mills theories are a common name for these kind of broad generalisations. According to Yang Mills theory flat space-time exists, whereas Kaluza Klein theory deals with a curved space-time. Riemannian manifolds of any dimension may be used as the base space of Kaluza Klein theory.

1.4 Higher Dimensional Cosmology

The existence of a five-dimension is an interesting possibility that may or may not be realized in nature. In particular, an extra dimension which behaves like space (instead of like time) but is compact rather than infinite features in many theories of particle physics. The extra dimension may for example explain properties of particles that we observe in our experiments. It may also predict the existence of other particles, or explain how gravity behaves. Whether the fifth dimension is "real" or not, it can provide a useful mathematical device. The five-dimension picture allows us to calculate things that are much harder to calculate in four-dimension. We will probably never directly see the five-dimension, but that does not mean we cannot find convincing evidence. For example, no one has ever detected an isolated quark, but the quark model does an incredible job at explaining the properties of larger, composite particles (hadrons). Because of this, scientists today agree that quarks are fundamental particles. Likewise, if we would collect enough evidence that five-dimension theories "work" explain the existing data and make successful predictions then the existence of a five-dimension would be very well motivated.

Einstein's theory comes from the interactions of the electromagnetic force, scientists have searched for over 100 years for ways to unite energy or light from the electromagnetic force with the other three forces, which are strong and weak nuclear forces and gravity. Two theories, independently developed and proposed by German mathematician Theodor Kaluza (Kaluza (1921)) and Swedish physicist Oskar Klein

(Klein (1926)) suggested the possibility of a fifth dimension where electromagnetism and gravity unify. Einstein's theory of relativity essentially suggested that space-time becomes warped, felt as gravity, by large objects like the Earth. He posited the measurement of gravitational waves and the possibility of black holes, though he spent his later years trying to disprove the idea of black holes, which scientists finally confirmed as real in 1971, decades after Einstein's death. But 100 years after he first published his theory of relativity, scientists also confirmed the existence of gravitational waves in September 2015, when scientists from the Laser Interferometer Gravitational-Wave Observatory first detected and measured gravitational waves that rippled through space when two black holes joined.

The claim that the cosmos has more than the four dimensions we can perceive that is, three spatial dimensions plus time is exotic enough. But the quest to prove that claim brings in a virtual menagerie of mysteries: mini-black holes and dark matter, gravitational waves and cosmic inflation, super-high-energy particle collisions and ultra powerful gamma-ray bursts. Even the physicists behind today's most talked about extra-dimensional theory, Harvard University's Lisa Randall and Johns Hopkins University's Raman Sun drum, aren't yet exactly sure whether the approaches will pay off. If there is gravity in our world, and if this gravity can only be explained by a fault in a higher dimension, then there must be a five-dimensional world (with 4 spatial dimensions and 1 time dimension) even if we don't see it or can't experience it.

1.5 The Cosmological Principle

The cosmological principle, which states that the universe is homogeneous and isotropic when viewed on a large scale, is the fundamental law of modern cosmology. Since the forces are expected to act uniformly throughout the universe, this should result in no observable irregularities in the large scale structuring. These two concepts have a distinct meaning in cosmology and are not interchangeable. Because the universe is homogenous and isotropic, there are no unique directions or locations inside it. Again, despite the fact that these two definitions seem to be identical, they both explain different aspects of the universe as a whole. For instance, if the universe is isotropic, then seeing in different directions won't change how the universe is structured. The universe seems the same to all witnesses when seen at the biggest scales, and it appears the same to a specific observer when viewed from all directions. When the universe is examined on the biggest sizes, homogeneity means that the average density of matter is roughly the same everywhere and that the universe is relatively smooth.

On tiny sizes, like the size of the Earth, the Solar System, or even the size of the Galaxy, it is obvious that this is not true. Only on extremely large sizes are phrases like "look the same" and "smooth in density" appropriate. In cosmology, we only take into account the isotropy and homogeneity of the universe on sizes of millions of light-years (a light-year is equal to around 1018 cm of space). Keep in mind that homogeneity for all observers is implied by isotropy for all observers (all locations in the universe). It is feasible to create worlds that are anisotropic yet homogenous, but not the other way around. Consider an observer who is surrounded by a matter distribution that seems to be isotropic; this means that the velocity field and other physical characteristics, such as the mass density, cannot have a preferred axis. The absence of a "centre" in an isotropic universe also signifies that it is isotropic. The north and south poles are created by the rotation of the Earth, yet the universe is visible from any angle.

When we think about the Big Bang, which is the origin of the universe, this is a crucial issue. The cosmic principle states that all physical rules apply to all situations. Distant stars, galaxies, and any other region of the universe all obey the same physical rules and models that govern our planet. Also keep in mind that it is presumpted that physical constants (such the gravitational constant, electron mass, and light speed) will remain constant throughout time and space.

Measurements of the cosmic microwave background provide the most convincing evidence in the contemporary era for the cosmological principle. In a nutshell, the CMB is a representation of the photons released from the early universe (we shall examine the CMB in a later lecture). Its haphazard look reflects its homogeneity and isotropy. The cosmological principle's most significant implication is that all of space is causally related to one another at some point in the past (although they may no longer be connected today). Thus, a homogenous universe supports the idea that the entire universe was created at a single instant of time. The universe is homogeneous and isotropic and has always been, according to the perfect cosmological principle, which we may prove by extending the cosmological principle through time. Therefore, it may be expected that objects we witness in the past would function according to the same rules of physics as they do now since the laws of nature remain constant.

1.6 Cosmological Models

Cosmological models are mathematical representations of the universe that make an effort to explain the universe's origin, evolution over time, current geometry and contents, current behaviour, and ultimate destiny. The predictions made by these models should be confirmed by further observations. These cosmological notions are based on Einstein's general theory of relativity. According to the general theory of relativity, field equations explain the connection between the curvature of space-time and the flow of energy-momentum. The solutions to the field equations give a chronology of the development of the cosmos. All cosmological theories are based on two fundamental presumptions known as "cosmological principles".

The most important cosmological model is the Cold Dark Matter (CDM) hypothesis. As cosmological constants, dark energy and cold dark matter are both accounted for in this universe model. It is sometimes referred to as the "Standard model" since it explains the CMB, large-scale structures, abundance of lithium, hydrogen, and helium as well as the universe's fast expansion. It is based on general relativity and the cosmological principle. This fundamental model of the cosmos is the basis for all other models of the universe.

1.7 Standard Model of Cosmology

According to the standard model of cosmology (SMC), in CDM model, universe formed in the "Big Bang" from nothing more than a hyper intense concentration of energy. The universe has about 26.7% dark matter, 5% ordinary matter, as well as 68.3% dark energy (DE). The standard model of particle physics (SMPP), and the general theory of relativity (GTR) (explains the physical universe as a whole) are the primary theoretical models upon which the SMC is built. It detects really large objects in classical mechanical contexts; thus, it differs depending on a number of ancillary generalizations. Four additional tenets of the SMC are as follows: (i) the universe formed in an energy-only Big Bang; (ii) ordinary matter accounts for 5% of the universe's mass-energy; (iii) the gravitational correlations between the first three tenets can be labeled as the GTR; (iv) the universe remains isotropic as well as on extremely vast scales. There is a widespread consensus that neither the SMPP nor the GTR are sufficient because they fail to account for key scientific results. SMPP does not account for the following: the occurrence of gravity, the existence of dark matter, such things mass hierarchy, etc. The GTR provides no explanation for inflation, Big Bang, and DE's origin.

SMC's current form, known as the CDM model, will be basically a modeling of the Big Bang cosmological model, postulating the presence of suitably massive dark matter particles, known as cold dark matter, and a cosmological constant coupled to DE. Yet, the only unaccounted for phenomena are dark energy as well as dark matter.

1.8 Dark Energy and Dark Matter

1.8.1 Dark Energy

The acceleration of the universe's expansion is attributed to a mysterious kind of energy known as dark energy. Scientists were taken aback when, in 1998, they discovered that the cosmos was expanding faster than expected owing to self-gravitation. Even though scientists have no idea what is accelerating the expansion of the cosmos, they now believe that there is some form of energy connected with a massive negative pressure. It was named "dark energy" (DE) by the researchers, which spreads over the cosmos in an even distribution. When it comes to dark energy, it has the opposite effect on structure formation as dark matter does, and opposes it, whereas dark matter has the opposite effect on structure creation. When space expands, density of dark matter drops, yet density of dark energy does not change. Consequently, more and more dark energy is generated as the space grows. Space-time may be a characteristic of the universe in a similar way to Einstein's theory that even empty space contains energy. Approximately 68.3 percent of the universe's mass is made up of dark energy (Ade et al. (2014)).

Astrophysics and modern cosmology's greatest challenge is to grasp the universe's late-time acceleration. For many academics and scientists in recent years, a focus on dark energy and new theories of gravity has helped them better comprehend how the universe works and how it may change in future.

1.8.2 Dark Matter

Dark matter is a hypothetical or enigmatic kind of matter that differs from conventional matter in that it does not emit, reflect, or absorb light or any other form of electromagnetic radiation. In other words, electromagnetic forces do not interact with dark matter. The interaction between it and conventional stuff, on the other hand, is purely gravitational. Galaxies are whirling at such a high rate of speed that the gravity created by their modest observable mass would have been ineffective in keeping them together. Long ago, these would have been shattered to pieces. This leads scientists to believe that the additional gravity required for galaxies to survive may be coming from an unseen source. "Dark matter" is the name given to this additional, imperceptible mass.

Fritz Zwicky's formal inference concerning the presence of dark matter was the first formal inference in the lengthy history of the dark matter theory (Zwicky (1937)). He examined the Coma cluster of galaxies in 1933. It was determined that Zwicky's estimate of the mass of Coma cluster was based on the total luminosity and number of galaxies in the cluster, but he found that the mass of the cluster was 400 times more than the visible mass. There must have been some non-luminous materials that had enough mass and gravity to keep the cluster together, so he said. It was he who coined the term "dunkle materie" for this mysterious substance.

In the 1960s and 1970s, Verma Rubin and Kent Ford used galaxy rotation curves to support the presence of dark matter (Rubin and Ford Jr (1970)). In most galaxies, dark matter accounts for six times as much mass as visible stuff.

The presence of dark matter may be inferred via gravitational lensing. According to the General Theory of Relativity, light bends when it comes into contact with a large object. It's like a magnifying glass for a large object: Since the overall mass of a galaxy or a cluster of galaxies can be measured via gravitational lensing, the presence of dark matter may be confirmed. It is now widely accepted that dark matter does exist.

According to the standard model, dark matter accounts for around 2678 of the total matter of the universe (Ade et al. (2014)) .

There may be both baryonic and non-baryonic components to dark matter. Some of dark matter is formed of baryons because it interacts with conventional matter gravitationally. However, the majority of scientists believe that dark matter is dominated by non-baryonic stuff.

Structure creation, galaxy formation, and the microwave background are all aided by dark matter. As density fluctuations condensed to create stars, galaxies, and galactic clusters, structure building started to take place following the Big Bang. Because radiation alters the density of ordinary matter, any structure formed by density perturbations would be impossible. There would have been no stars or galaxies if the cosmos had simply contained ordinary substance. However, it was the dark matter that rescued the day. Because radiation has no effect on its density disturbances, it aids in the creation of structures by accelerating the process of compressing ordinary matter.

Three forms of dark matter have been proposed:

Hot dark matter (HDM) has a very little mass, almost zero, and its particles travel with ultra-relativistic velocity exceeding 0.95c, close to the velocity of light associated with a high energy state; hence, HDM cannot sustain structure development. Because neutrinos have a mass smaller than that of an electron, they may make up the HDM, as neutrinos can only interact with conventional matter through gravity and weak force.

Warm dark matter (WDM) is heavier than HDM and its particles travel at relativistic speeds ranging from 0.1c to 0.95c. Due to its fast speed, WDM, like HDM, does not allow for the creation of structures. Weakly interacting neutrinos, such as the sterile neutrino, might be used in WDM.

Cold dark matter (CDM) is the heaviest of the three types of dark matter, and its particles move at a velocity of less than 0.1c, either sub or non-relativistic. We don't know what makes it up. In addition to gigantic compact halo objects, like as black holes, new particles like axions may also be contenders for the title of candidate. According to A. Peter (Peter (2012)), new particles are the only viable contenders for dark matter. The CDM, according to the majority of scientists, was responsible for the emergence of structure.

1.9 Hubble's Law and Hubble's Constant

There is a relationship between the recession velocity and the distance between galaxies revealed by Hubble's observations: As a result, Recessional velocity is equal to Hubble's constant times the distance.

i.e.

$$V = HD$$

where V is the galaxy's measured velocity, generally in kilometers per second. H is Hubble's constant in km/sec/MPc. MPc (milliparsecs) is the distance between the galaxies. However, the Hubble's constant H must be measured precisely before the rule may be employed in practice. There are inherent errors in the H calibration. The distances to galaxies with strong red-shifts must be determined using separate techniques. The galaxy's random speed must be dwarfed by the recessional speed. According to current research, the Virgo Cluster of galaxies has been shown to be most suited for the task. A huge number of brilliant galaxies and a distance of $(m - M \cong 31)$ make photometric distance measurement possible on the one hand, while red-shifts are also important in this region. To make matters worse, the Virgo Cluster's random velocities towards the centre of the mean recessional motion are on par with the mean recessional motion's own random velocities. The rate of cosmic expansion is determined by the Hubble's parameter or Hubble's constant H, which we may deduce from the preceding equation. $H = \frac{V}{D}$, V is the velocity of an object at a distance D, which is the item's distance from the source. Also the scale factor a is the logarithmic derivative of $a(t)$

i.e.

$$H = \frac{\dot{a}}{a}$$

.

The calculation of the mean recessional speed of the cluster is based on the movements of the individual members of the cluster, each of which has a substantial random motion. Because so much time and effort has gone into the accurate assessment of H, this value should be taken with a caution.

1.10 Some Important Cosmological Parameters

1.10.1 Cosmological Constant

The cosmological constant was initially included into field equations by Einstein. As a mathematical fix to the theory of general relativity, Lambda (Λ) is often used as a symbol. General relativity, in its simplest form, predicted that the universe will either expand or shrink. He created this new phrase because he believed the cosmos was static and wanted to halt its growth. After realising this was an unstable solution, a Russian mathematician named Friedmann presented an expanding universe model, which is now known as the big bang theory. Einstein viewed the cosmological constant phrase as his "biggest error" when Hubble's analysis of neighbouring galaxies confirmed that the universe was expanding.

On theoretical reasons, several cosmologists argued for the revival of the cosmological constant phrase. Vacuum energy density is referred to as this phrase in modern field theory. The inclusion of a cosmological constant factor has major consequences for particle physics and our understanding of nature's basic forces, since it would be necessary for this energy density to be equivalent to that of other types of matter in the universe. Because the cosmological constant phrase considerably increases the agreement between theory and observation, it is the most appealing aspect of this universe.

In general, the big bang expansion may be seen in the gravitational pull of the universe stuff. Astronomers have only lately been able to see the supernova explosions, which are the deaths of massive stars, to determine how fast the universe is expanding. Preliminary as these findings are, they suggest that the cosmos may include a strange sort of matter or energy that acts as a gravitational attractor instead of a magnet. In the cosmological constant, this energy may be seen in action. There is still a great deal of work to be done in order to solve this riddle. Numerous additional findings point to the need of a cosmological constant. As an example, the projected age of the universe is significantly bigger if the cosmological constant now accounts for most of the universe's energy density. The addition of a new word to the standard model: cosmological constant. Large-scale galaxy and cluster distributions, changes in the cosmic microwave background and X-ray characteristics all seem to be compatible with a model derived from big bang theory.

The space-time in Einstein's general theory of relativity is described by the pseudo-Riemannian metric as

$$ds^2 = g_{ij}dx^i dx^j; i, j = 1, 2, 3 \text{ and } 4 \tag{1.1}$$

where the symmetric metric tensor g_{ij} is gravitational potential. The gravitational field equation or, simply the field equations in Einstein's general theory of relativity are given by

$$G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij} = -\frac{8\pi G}{c^4}T_{ij} \tag{1.2}$$

where G_{ij} is the Einstein tensor, R_{ij} is the Ricci tensor, R is the Ricci scalar (Scalar Curvature) and T_{ij} is the energy-momentum tensor due to matter and Λ is the cosmological constant. In order to study Static cosmological model, Einstein modified his field equation by introducing another term as

$$R_{ij} - \frac{1}{2}Rg_{ij} + \Lambda g_{ij} = -8\pi T_{ij} \tag{1.3}$$

where Λ is the cosmological constant. Later, he later dropped this term by saying that "greatest blunder" of his life. In both the equations (1.2) and (1.3), left hand sides represent the geometry of space and the right hand sides represent the matter.

1.10.2 Deceleration Parameter

Deceleration parameter (q) is a dimensionless quantity is used to estimate the cosmic acceleration of the expansion of our universe. It is denoted by q and defined as

$$q = -\frac{a\ddot{a}}{a^2}$$

An average scale factor known as a is used to express derivatives with respect to appropriate time. When q is negative, the cosmos accelerates its expansion. This is because q may be either positive or negative depending on whether a is positive or negative. When the deceleration parameter q was first defined, it was assumed that its value would be positive. But present day' s observational data, it has been established that the universe is accelerating with expanding instead of showing down as predicted by the Big Bang theory (Silk (1989)). We think that the cosmos is dominated by dark energy because it has an abundance of positive energy density and negative pressure that contributes to its late-time acceleration.

1.10.3 Hubble's Parameter

It is recognized as the Hubble's constant or Hubble's parameter that v=Hd, which is the speed at which galaxies at a distance d from us are receding from us. The cosmic time affects the Hubble's parameter H. In this case, the scale factor is a(t), then $a(t) = \frac{d(t)}{d_0}$, which gives $\frac{\dot{a}}{a} = H$.

The generalized signify Hubble's parameter for five dimensional anisotropic Kaluza-Klein space-time is given by

$$H = \frac{1}{4}(\frac{3\dot{A}}{A} + \frac{\dot{B}}{B}) \tag{1.4}$$

The directional Hubble's parameters H_x, H_y, H_z and H_ϕ in the direction of x, y, z and ϕ respectively for the Kaluza-Klein metric are

$$H_x = H_y = H_z = \frac{\dot{A}}{A}$$

and

$$H_\phi = \frac{\dot{B}}{B}$$

1.10.4 Equation of State Parameter

In cosmology EoS (equation of state) parameter of a perfect fluid is characterizes the models of dark energy usually, by defining the EoS parameter ω is a dimensionless number is equal to the ratio of its pressure p to its energy density ρ of a cosmic material i.e.

$$\omega = \frac{p}{\rho} \tag{1.5}$$

It's a critical tool for studying the evaluation and ultimate destiny of universe. The EoS parameters for barotropic fluid are as follows: $\omega = 1$ for stiff matter, $\omega = \frac{1}{3}$ for radiation, $\omega = 0$ for matter dominated universe and $\omega = -1$ for vacuum dominated universe (negative pressure).

It is currently believed that at present observation the value of equation of state parameter ω close to -1, while if $\omega > -1$, then this is called quintessence, and if $\omega < -1$ then it is called phantom model respectively (Knop et al. (2003)), (Tegmark et al. (2004)). Recent observational limits constrain the equation of state parameter to the interval $-1.67 < \omega < -0.62$ while a combination of data from the observations of SNIa, CMB, anisotropy and galaxy clustering gives the limit on ω as $-1.33 < \omega < -0.72$.

1.10.5 Scale Factor of the Universe

To indicate how the universe size increases throughout time in relation to our current time, the scale factor (a) is a dimensionless quantity that is commonly symbolised by the symbol t, which stands for cosmic time. As a result, the scale factor is a measure of the universe expansion.

If $d(t)$ is the gap between two celestial objects, such as two stars, and their current distance t, then $a(t) = \frac{(d(t))}{d_0}$.

At the same time, where z is the red-shift of an object's light, then $a(t) = \frac{1}{z} + 1$

1.11 Strings and String Cosmology

The origin and evolution of our universe remain one of the greatest scientific mysteries, despite the fact that the Big-Bang hypothesis is the most commonly accepted explanation for how the world came into being and evolved. Understanding the universe's true physical condition at this early time is crucial. To explain the events that occurred in the early stages of the creation of our universe, cosmologists employ the concept of string theory, in which cosmic string is one of the most important study objects. Cosmologists use string theory to look into the period of time before the universe's initial particle was produced. Our cosmos may have undergone a number of phase transitions when it cooled from its initial state of high temperature after the Big Bang (i. e. the universe passes through its critical temperatures). It's possible that the symmetry of the cosmos was inadvertently broken during the phase change. The phase transitions in the early stages of our universe may have caused a variety of topological defects, including cosmic strings, domain walls, monopoles, and textures (Kibble (1976); Mermin (1979)). A defect might be a string, a domain wall, a monopole, or any other topological discontinuity depending on the vacuum topology.The topological flaws, according to Pando and his co-authors (Pando et al. (1998)), are what created the structure of the cosmos. Only the topological imperfection known as the string can explain such exciting cosmic consequences as galaxy formation and the double-quasar problem. According to (Vilenkin (1981); Vilenkin (1985); Vilenkin and Shellard (1994)) the string theory is likewise a rival for the unification of all forces. Our universe's existing structures are consistent with a vast network of strings that existed in the early phases of our universe. Strings are one of the main sources of the density perturbations that are necessary for the large-scale structure of our universe. It's possible that a gravitational field was produced by the tension energy of the threads.As a result, since they are an interesting scientific topic, the gravitational effects of the strings are also investigated. The general theory of relativity was where strings were first introduced (Letelier (1979); Letelier (1983)).

A significant difficulty in modern cosmology is understanding how the universe's structure develops, specifically how galaxies and other large-scale structures are born. Existing theories for the creation of the universe's structure include quantum-fluctuation amplification in a scalar field during inflation and a symmetry-breaking phase shift in the early cosmos that produced topological defects. Topological flaws during the symmetry breakdown are a predictable outcome due to unification theory. Among other defects, cosmic threads have been shown to be the most helpful for creating cosmic structures. However, inflation as a realistic structure formation model has demonstrated to be computationally far more challenging to produce solid predictions with which to handle data. Understanding cosmic strings as the origins of large scale structure and cosmic microwave background anisotropies has just recently made significant strides. Cosmic strings are strands of energy because of the early universe's symmetry-breaking phase shift. By attracting adjacent materials via gravitational interactions, they act as a starting point for the formation of structures. In relativity, as light travels by a string, the geodesic path of light may be observed bending in the direction of the string.

The matter and energy density of the cosmos are disturbed as the scale of strings goes from microscopic to large. According to research by Zel'dovich (Zel'dovich (1980)) and Vilenkin (Vilenkin (1981)) there is evidence to suggest that the creation of galaxies and galaxy clusters may be owing to gravitational effects induced by the early universe's network of cosmic strings. To make a long string, particles are connected to a geometric thread. The cloud is a generalisation of Takabayashi's realistic string model. The elimination of strings might possibly result in a cloud of particles. Given that we cannot yet observe the strings, this is a crucial aspect of a string cloud model for cosmology.

When the temperature dropped below specific threshold temperatures during the early universe phase transition, stable topological structures called strings were created. A vast scale network of strings may have existed in the early phases of our cosmos, according to observations made of it using modern scientific methods. Strings that contain stress-energy include geometric and enormous strings. The anisotropy of the cosmos is caused by the existence of strings, but they are no longer observable. These strings can produce a wide range of intriguing astrophysical findings rather than harming the cosmological models. The nature and basic structure of the early universe can also be characterised by strings. This provides us a single theoretical framework in which all matter and forces are combined. due to the crucial role strings play in characterising the development of the early universe.

Considering that the massive strings were formed from the massless geometric strings with particle attached along its extension, Letelier (Letelier (1979)) formulated the equation of energy momentum tensor for a cloud of massive strings as

$$T_{ij} = \rho \nu_i \nu_j - \lambda x_i x^j \tag{1.6}$$

where ν_i and x_i satisfy the conditions

$$\nu^i \nu_i = -x^i x_i = -1, \nu^i x_i = 0 \tag{1.7}$$

Here ρ is the rest energy density for a cloud of strings with particles attached to them.

$$\rho = \rho_p + \lambda,$$

ρ_p being the rest energy density of particles attached to the strings and λ the tension density of the strings. Here p and ρ are a function of cosmic time t only. x_i is a unit space-like vector instead of the direction of strings so that $x^2 = x^3 = x^4 = 0$ and $x^1 \neq 0$.

1.12 Conharmonic Transformation

Conharmonic transformations are a particular class of conformal transformations that preserve the harmonicity of functions. It is generally known that this category of transformations has a tensor invariant, sometimes known as a conharmonic curvature tensor, which exhibits the Riemannian curvature tensor's traditional symmetry features. If the metrics of two Riemannian spaces g_{ij} and g^*_{ij} meet the relation established by the equation, then those spaces are said to be conformal spaces.

$$g_{ij} = e^{-\phi} g^*_{ij}, \tag{1.8}$$

where ϕ is a real valued function of coordinates and the correspondence between g_{ij} and g^*_{ij} are called conformal transformation (Ahsan (2008)). A function with Laplacian vanishing is referred to as harmonic function. Ishii (Ishii (1957)) have been studied the condition under which the harmonic functions remain invariant, who introduced the conharmonic transformation as a subgroup of the conformal transformations satisfied the following relation

$$\phi^i_{;i} + \phi_{;i}\phi^i_{;} = 0, \tag{1.9}$$

where the covariant derivative with respect to metric g_{ij} denoted by $(;)$.

A rank four tensor L^l_{ijk} that retains its invariant form under conharmonic transformation for an n-dimensional Riemannian differentiable manifold is given by

$$L^l_{ijk} = R^l_{ijk} - \frac{1}{n-2}(g_{ij}R^l_k - g_{ik}R^l_j + \delta^l_k R_{ij} - \delta^l_j R_{ik}), \tag{1.10}$$

where R^l_{ijk} and R_{ij} is the Riemannian curvature tensor and the Ricci tensor respectively. Equation no. (1.10) is known as Conharmonic curvature tensor. A space time in which R^l_{ijk} vanish at each point is called conharmonically flat space time. Conharmonic curvature tensor shows the deviation of the space time from conharmonic flatness.

Einstein's space has a constant curvature and is conharmonically flat space-time. Cosmology is extremely well aware of the importance of the spaces with constant curvature. The simplest cosmological model of the universe is produced if we assume that the cosmos is isotropic and homogeneous, which is known as the cosmological principle.

1.13 Energy Momentum Tensor for Perfect Fluid

The significance of perfect fluid is a fluid that is frictionless, homogenous, and incompressible and is incapable of enduring any tangential stress or action in the form of shear, however, the normal force still operates between the adjacent layers of fluid. A perfect fluid, whether it is at rest or in motion, has an equal pressure in all directions at every point. The energy stresses in the rest frame, viscosity, and heat conduction can all be further used completely characterize it.
The energy-momentum tensors characterizing matter are given by

$$T_{ij} = \rho u_i u_j + S_{ij} \tag{1.11}$$

where ρ symbolizes the mass density, u_j symbolizes the four particle velocity vectors $u_i = \frac{dx_i}{ds}$, and S_{ij} symbolizes the stress tensor, and $c = 1$ is the speed of light. Unless the matter consists of a perfect fluid, viz, one whose pressure is isotropic the stress tensor can be affirmed as

$$S_{ab} = p(u_i u_j - g_{ij}) \tag{1.12}$$

where p symbolizes the pressure.
Therefore the energy-momentum tensors can become

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij} \tag{1.13}$$

The only stress individuals can sustain is the isotropic pressure p, the mass density is indicated by ρ. The pressure of the sources can generally be neglected, except in the early universe. A perfect fluid to zero pressure is theoretically related to particles that on the substrate surface. Whereas any arbitrary displacement would result in pressure. For the early universe meanwhile, uniform radiation is considered to have strongly influenced.

1.14 The Jerk Parameter (j)

The Jerk Parameter in cosmology is defined as the dimensionless third derivative of the scale factor with respect to cosmic time t (Visser (2004), Sahni et al. (2003)). It is defined as

$$j(t) = \frac{1}{H^3}\frac{\ddot{a}}{a} = \frac{(a^2H^2)^{//}}{2H^2} \tag{1.14}$$

where over "dots" and primes denote derivatives with respect to cosmic time and the scale factor respectively.

The jerk parameter (j) appears in the fourth term of a Taylor expansion of the scale factor around a_0.

$$\frac{a(t)}{a_0} = 1 + H_0(t-t_0) - \frac{1}{2}q_0H_0^2(t-t_0)^2 + \frac{1}{6}j_0H_0^3(t)^3 + 0\left[(t-t_0)^4\right] \tag{1.15}$$

In equation (1.15) can be written as

$$\dot{j}(t) = q + 2q^2 - \frac{\dot{q}}{H} \tag{1.16}$$

where H is the Hubble parameter, q is the deceleration parameter and overhead dot denotes the derivatives with respect to cosmic time t.

1.15 Statefinder Parameter (r, s)

Several theoretical and experimental evidence support the ΛCDM model, in which the cosmological constant problem plays a vital role of dark energy model in general theory of relativity. The most recent iteration of our knowledge about the creation of the universe is the ΛCDM model. The ΛCDM model is regarded as a standard model of cosmological at the recent phase. In an effort to distinguish between various dark energy models. Sahni et al. (Sahni et al. (2003)) created a pair parameters (r, s), called statefinder parameters, which are essential in cosmology.

The statefinder parameter (r, s) defined as follows

$$r = \frac{\dddot{a}}{aH^3} = 1 + 3\frac{\dot{H}}{H^2} + \frac{\ddot{H}}{H^3} \tag{1.17}$$

and

$$s = \frac{r-1}{3(q-\frac{1}{2})} \tag{1.18}$$

where a is the cosmic scale factor, H is the Hubble parameter, q is the deceleration parameter and overhead dot denotes the derivatives with respect to cosmic time t.

1.16 Review of some work related to Five Dimensional Kaluza-Klein Space-Time

The study of higher dimensional space-time is an active field of research to expose how the gravity and the electromagnetism can be unified from Einstein's field equations generalized to five dimensions. In a certain sense the Kaluza-Klein theory resembles ordinary gravity, except that it is inscribed in five dimensions instead of four. This theory has been regarded as a candidate of fundamental theory due to the possible work of unifying the fundamental principle. This fact has attracted many prominent authors studied the five dimensional Kaluza-Klein space-time in the framework of general theory of relativity.

According to Chodos and Detweiler (Chodos and Detweiler (1980)), the Einstein-Maxwell equations still hold true in a cosmology with 3+1 observable dimensions when the vacuum field equations of general relativity are solved in 4+1 space-time.

In the context of Kaluza-Kiein theories, Marciano (Marciano (1984)) investigated how the basic constants are affected by time. His investigation revealed correlations between low-energy couplings and masses, which led him to conclude that evidence for additional spatial dimensions may be found through changes over time in any of these parameters.

Kim and Cho (Kim and Cho (1987)) demonstrated how, in the modified Brans-Dicke theory containing a torsion field in the 5-dimensional Kaiuza-Klein space-time, the electromagnetic field and scalar field develop during the reduction of five-dimensional (5D) action.

Within the context of the 5D Kaiuza-Klein theory, Basu and Ray (Basu and Ray (1989)) found the whole solution set for the five-dimensional vacuum Einstein equations.

The static, spherically symmetric vacuum cosmological model was studied by Chatterjee (Chatterjee (1990)) in relation to the Wesson five-dimensional variable gravity theory.

By assuming that the metric coefficients indicate a functional connection, Chatterjee and Bhui (Chatterjee and Bhui (1990)) simulated the five-dimensional field equations based on Wesson's variable gravity theory paired with an energy-momentum tensor including a viscous fluid.

In an inhomogeneous Kaiuza-Klein space-time, Chatterjee (Chatterjee (1993)) solved the Einstein field equations using enormous strings as the source.

In a 5D Kaiuza-Klein space-time, Chatterjee and Banerjee (Chatterjee and Banerjee (1993)) were able to get precise cosmological solutions to the field equations for an inhomogeneous fluid distribution in the form of dust.

A Kaiuza-Klein kind of inhomogeneous cosmological model with a matter field in the form of dust was constructed by Chatterjee et al.(Chatterjee et al. (1994)) .

Rahaman and Bera (Rahaman and Bera (2001)) investigated the physical behaviour of the cosmological models in perfect fluid and vacuum and in the context of Kaluza-Klein cosmology inside the LG framework.

In the context of general relativity, Rahaman and Kalam (Rahaman and Kalam (2002)) investigated the gravitational field of spherically symmetric domain walls in the 5D space-time.

Five-dimensional string cosmology models for geometric string, Takabayasi string, and barotropic equation of state for strings based on LG were built and investigated by

Rahaman et al.(Rahaman et al. (2003a)) .

Within the context of LG, Rahaman et al. (Rahaman et al. (2003c)) were able to find an accurate solution of the field equations for a thick domain wall in a five-dimensional Kaluza-Klein space-time. They demonstrated the absence of a particle horizon in the domain wall and the attractiveness of the domain wall's gravitational pull. They also discovered that space-time exhibits nonsingular spatial and temporal dynamics. Also in LG, Rahaman et al. (Rahaman et al. (2003b)) constructed homogenous ideal fluid cosmological models in five dimensions. Additionally, they looked at the models' dynamical behaviour when the displacement vector was constant.

Five-dimensional homogeneous cosmological models with a changeable gravitational constant and bulk viscosity in LG were created and investigated by Singh et al.(Singh et al. (2004)). They noticed that the model findings are within the range of the data.

According to an equation developed by Khadekar et al. (Khadekar et al. (2004)), the mass is related to an integral over the gravitational force in 5D space-time.

In general relativity, Bhui et al.(Bhui et al. (2005)) constructed a 5D homogeneous cosmological model under the assumption that the equation of state is time dependent. They discovered that when the additional spatial dimension shrinks, the entropy of the visible three-dimensional space increases.

Using field-theoretic energy momentum tensors for monopole configuration, Rahaman and Bhui (Rahaman and Bhui (2005)) investigated the gravitational field of a non-stationary global monopole in five dimensions. They demonstrated the attractive gravitational effects that the monopole had on the test particles.

A 5D cosmological model with a perfect fluid source and variable Λ in general relativity was studied by Khadekar and Samdurkar (Samdurkar (2005)).

In an attempt to build a five-dimensional perfect fluid cosmology model using the Lyra manifold, Mohanty et al.(Mohanty et al. (2006)) discovered that neither perfect fluid nor dust distributions survive. Finally, the vacuum field equations' precise solutions are discovered.

In order to study the dynamics of the model, Rahaman et al. (Rahaman et al. (2006)) built a bulk viscous cosmological model in general relativity under the assumption of a time-dependent equation of state in 5D KK spacetime.

Using a perfect fluid energy momentum tensor, Venkateswarlu and Pavan Kumar (Venkateswarlu and Kumar (2006)) solved the field equations of the Barber SSC theory of gravity for 5D FRW space-time. They demonstrated that the field equations in this theory permit vacuum solutions when the pressure in the fifth dimension is zero.

In the framework of general relativity, Yilmaz (Yilmaz (2006)) have researched KK cosmic solutions in higher dimensions for quark matter coupled with string cloud and domain barriers.

In a scalar tensor theory of gravity provided by Saez and Ballester (1986) for a five dimensional Kaluza-Klein space-time, Reddy and Naidu (Reddy and Naidu (2007)) built string cosmological models and examined some of the models' characteristics.

Based on Lyra geometry, Mohanty et al. (Mohanty and Mahanta (2007b)) were able to derive accurate solutions to the five-dimensional vacuum cosmological field equations, and they demonstrated that the model does not maintain either the dust distribution or the ideal fluid distribution.

In a five dimensional space time, Mohanty et al. (Mohanty et al. (2007)) shown that the universal ideal fluid distribution does not persist and degenerates into stiff fluid distribution based on the Lyra manifold. As a result, Lyra manifold is used to build five dimensional vacuum and stiff fluid models of the cosmos.

In the context of the second self creation theory of gravity, Mohanty and Mahanta (Mohanty and Mahanta (2007b)) built five-dimensional anisotropic homogeneous cosmological models and demonstrated how, under the power law, ideal fluid models degenerate into Zelodovich fluid models.

Brans-Dicke theory of gravity was used by Reddy et al. (Reddy and Naidu (2007)) to develop the Kaluza-Klein string cosmological model, and they explored some of the model's physical characteristics.

The five-dimensional vacuum universe in general theory of relativity is the outcome of one case, according to Mohanty and Mahanta's (Mohanty and Mahanta (2007a)) constructed a five-dimensional axially symmetric string cosmological model in the Lyra manifold, while the other case produces a string cosmological model in the Lyra manifold. They found that the string's total tension density and rest energy density disappear in the cosmic string model.

Khadekar et al. (Khadekar et al. (2008b)) investigated string dust cosmological models with particles attached to them by considering three different forms of variable Λ in the context of five-dimensional KK space-time.

Robertson Walker cosmology model of the Kaluza-Klein type was created by Khadekar et al. (Khadekar et al. (2008a)) and includes the dynamical cosmological component Λ.

Khadekar and Shelote (Khadekar and Shelote (2009)) constructed higher dimensional cosmological model of the universe by assuming different form of variable cosmological constant term Λ. They also studied the dynamical behaviors of the model for the gamma law equation of state.

Khadekar and Kamdi (Khadekar and Kamdi (2009)) investigated 5D KK cosmological model of universe are consider with variable cosmological constant term in presence of perfect fluid. A unified description of early universe has been presented in which an inflationary phase is followed by radiation-dominated phase in the context of Kaluza-Klein theory of gravitation.

Tiwari et al. (Tiwari et al. (2010)) constructed five dimensional Kaluza-Klein space-time in presence of perfect fluid source with variable G and Λ. By using a relation between the metric potential and an equation of state they found that the universe is expanding.

Ozel et al. (Ozel et al. (2010)) analyzed the Kaluza-Klein type Robertson Walker (RW) cosmological model with variable cosmological term Λ in the presence of strange quark matter. They found that vacuum energy density may have negative pressure. In this case quarks and gluons which are confined are moving freely. They are not moving collectively, i.e. as a perfect liquid.

In higher dimensional cosmology, Khadekar and Kamdi (Khadekar and Vaishali (2010)) obtained the exact solution of the Einstein field equations by assuming the global equation of state parameter $p = \frac{1}{3}\phi\rho$ in presence of Λ and G. They studied the field equations for perfect fluid cosmology are identical to Einstein equations for G and Λ.

Sharif and Khanum (Sharif and Khanum (2011)) studied 5D Kaluza-Klein cosmology variable G and Λ. They demonstrated that the field equations in this theory when the pressure and energy density in the fifth dimension are zero.

Adhav et al. (Adhav et al. (2012)) investigated Kaluza-Klein interacting cosmic fluid cosmological model dominated by two interacting perfect fluid components during the expansion. To obtained the exact solutions of the field equations they considered barotropic equations of state for pressure and density.

Reddy et al. (Reddy et al. (2012)) constructed 5D Kaluza-Klein Cosmological Model in $f(R,T)$ theory of Gravity. The found that the model in f (R,T) theory of gravity has stability and has no initial singularity.

Jain et al. (Jain et al. (2013)) implications of time varying cosmological constant on Kaluza-Klein cosmological model. They demonstrated the present cosmological model is nonsingular model.

Naidu et al. (Naidu et al. (2013)) constructed 5D KK bulk viscous string cosmological model in Brans-Dicke scalar-tensor theory of gravitation when the source of energy momentum tensor is a bulk viscous fluid containing one dimensional cosmic strings.

Rokde and Pund (Rokde and Pund (2014)) studied 5D Kaluza-Klein type of cosmological model of universe with dynamical cosmological term Λ. They have established a relationship between Cosmic matter and vacuum energy density parameters for flat universe.

Oli (Oli (2014)) constructed cosmological models with G and Λ in a five-dimensional Kaluza-Klein space-time. They found in which the gravitational constant (G) increases and cosmological constant (Λ) decreases with respect to cosmic time t.

Katore et al. (Katore et al. (2014)) constructed spatially homogeneous and anisotropic Kaluza-Klein Dark Energy cosmological model with magnetized fluid in the scalar tensor theory of gravitation. They obtained exact solutions of the models by using Volumetric exponential and power law expansion.

Khadekar et al. (Khadekar and Wanjari (2015)) constructed KK type cosmological model with Ω–dependent cosmological constant and big rip singularity. They shown that the big rip singularity occurs at finite time $t = t_s$ by violating NEC, WEC, SEC and satisfying the DEC for $\alpha > 0$.

Jain et al. (Jain and Bhoga (2015)) investigated KK type cosmological models with time varying G and (Λ) in the presence of bulk viscous. They studied through the variation of decelerating parameter q with cosmic time, both cosmological models show that the universe is accelerating but at the early stage of the universe the behaviour of both models is quite different.

Raut et al. (Raut et al. (2015)) constructed KK anisotropic dark energy cosmological model with special form of deceleration parameter. They studied that the spherical symmetry of the cosmos obtained during inflation has not been altered in later eras of the universe, even if it turns out that an anisotropic DE does not necessarily disrupt the symmetry of space.

Ghate (Ghate (2016)) investigated KK cosmological model with variable EoS parameter in general relativity when the universe is dominated by dark energy.

Sahoo et al. (Sahoo et al. (2016)) investigated a class of KK cosmological models in $f(R, T)$ theory of gravity with $\Lambda(T)$. They found that the exponential volumetric expansion model exhibits behaviour that is more similar to a lambda cold dark matter model.

Sahoo (Sahoo (2017)) discussed KK universe with wet dark fluid (WDF) source in $f(R; T)$ gravity. They found that the model starts at Big Bang and has a point type singularity.

Pawar et al. (Pawar et al. (2018)) discussed KK string cosmological model in $f(R, T)$ theory of gravity.

Khadekar et al. (Khadekar and Ramtekkar (2019)) studied the behavior of cosmological parameters for particular and arbitrary values of n, m and α in Kaluza Klein Type FRW Cosmological Model With Extended Chaplygin Gas.

Hatkar et al. (Hatkar and Katore (2020)) investigated Kaluza-Klein Cosmological Models with Polytropic Equation of State in Lyra Geometry.

Singh et al. (Singh et al. (2021)) discussed the essence of $f(R, T)$ gravitation theory in five dimensional universe and see the role of dark energy in the form of wet dark fluid in a Kaluza-Klein universe. They found that the dark energy is not exaggerated in contributing to the accelerating expansion of the universe though the expansion is inherent as a result of the theory itself and due to the geometric contribution of matter.

1.17 Objectives:

- To investigate dark energy cosmological models in five dimensional Kaluza-Klein space-time with different theories of gravitation in presence of different types of fluid like perfect, bulk viscous etc..

- To study the string cosmological models in five dimensional Kaluza-Klein space-time in presence of time dependent deceleration parameter.

- To investigate the source and nature of dark energy cosmological models in five-dimensional Kaluza-Klein space-time with special form of deceleration parameter.

- To study the five dimensional Kaluza-Klein cosmological models with the variable cosmological and gravitational constants in conharmonically flat space-time.

1.18 Methodology and Tools:

In this research work, all the data are used secondary data. The secondary data are collected from different sources. The main source of secondary data are collected from Internet, websites, journals, library, university etc.

This thesis is entirely theoretical. In this thesis, we focus on higher-dimensional space-time, the Einstein field equation, various form of deceleration parameter, equation of state parameters, etc. By plugging these values into the field equations, we constructed a set of nonlinear differential equations.

The problems are solved manually and considering the different cases and compared the solutions obtained with the present observational data to check our solutions are realistic or not.

Tensor algebra, ordinary and partial differential equations, integration, and mathematical software are the primary methods used to solved the mathematical problem and to draw the required graph in this research work.

Finally, we prepared the documents containing the final results of our research using the Microsoft Office and Latex typesetting systems.