Chapter 3

Anisotropic Cloud String Cosmological Model with Five Dimensional Kaluza-Klein Space-Time

3.1 Introduction

Now a days string cosmology has recently received a lots of attention, due to its crucial contribution to the investigation of the origin and evolution of the physical universe prior to the creation of particles. Cosmologists are interested in the field of string cosmology to study the mysterious phenomena that have yet to be observed and investigate the unseen information of physical universe. For this reason, cosmologists are extremely interested to understand the past, present, and future evolution of the universe. However, as of right now, we don't have enough solid data to draw certain conclusions on its origin and evolution. So, further inquiry is required to discover the enigmatic occurrences of the entire universe. Stachel (1980) and Letelier (1983) initiated the study of strings within the framework of general relativity, since the string perfectly captures the early stages of the formation of the universe. It is believed that strings cause density perturbations that result in the formation of massive scale

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structures in the universe (Zel'dovich et al. (1974); Zel'dovich (1980)), which has attracted the attention of many eminent authors working in the field of cosmic strings within the framework of general relativity (Kibble (1976); Kibble (1983)).

General relativity's paradigm has been used to explore Kaluza-Klein cosmic solutions for quark matter linked with domain barriers and string clouds in higher dimensions by Yilmaz (2006) and Adhav et al. (2008). Reddy et al. (2007) and Reddy and Naidu (2007) investigated several theories of gravitation for the string cosmological model in higher-dimensional space time. In the setting of 5D Kaluza-Klein space-time, Khadekar et al. (2008b) examined string dust cosmological models with particles attached to them by taking into account of three different kinds of variable Λ. In higher dimensional space-time, Khadekar et al. (2007) investigated a string cosmology model with bulk viscosity. Nimkar (2017) discussed how the electromagnetic field affects with the string cosmological model in general relativity. The Kaluza-Klein string cosmological model has been discussed by Pawar et al. (2018) within the context of the f(R, T) theory of gravity. Higher dimensional Bianchi type-I cosmological model were investigated by Krori et al. (1994). They found evidence that string and matter coexisted throughout the universe's development. Scale covariant theory of gravity has been used by Venkateswarlu and Pavankuma (2005) to study a string cosmological model in higher dimensional space-time. Rahaman et al. (2003a) attained the precise solutions of the field equations for the space-time with high dimensions in the Lyra manifold structure where the source of the gravitational field is a heavy string. By assuming that the product of the energy density and tension density is zero, Kandalkar et al. (2012) constructed Bianchi type-III string cosmological models with magnetic fields in the context of general relativity and obtained exact solution of the field equations. In the general theory of relativity, Mohanty and Samanta (2009) studied a 5D axially symmetric string cosmological model in the presence of bulk viscous fluid. Samanta and Debata (2011) developed a Bianchi type-I 5D string cosmological model in the Lyra manifold. Some of the distinguished authors who investigated various string cosmological models in higher-dimensional space time within the contexts of general relativity are Choudhury (2017); Tripathi et al. (2017); Dubey et al. (2018); Tiwari et al. (2019); Ram and Verma (2019); Mollah et al. (2019); SINGH and BARO (2020); Mollah and Singh (2021) and

Baro et al. (2021).

In this chapter, we have investigated 5D Kaluza-Klein string cosmological models in presence of variable deceleration parameter. This chapter is organised as follows: Section 3.2 is devoted to the metric and Einstein's field equations. We discussed the solution and theoretical calculations of the field equations in Section 3.3. In section 3.4, the physical interpretation and its behaviour has been provided.

3.2 Metric and Field Equations

A spatially homogeneous and anisotropic 5D Kaluza-Klein space time has been considered and it is as follows

$$ds^2 = dt^2 - A^2(dx^2 + dy^2 + dz^2) - B^2 d\psi^2 \tag{3.1}$$

where A and B are functions of cosmic time t only and the fifth coordinate ψ is taken to be extended space like coordinate.

The Einstein's field equation is given by

$$R_{ij} - \frac{1}{2} R g_{ij} = -T_{ij} \tag{3.2}$$

where R_{ij} is the Ricci tensor R is the Ricci scalar g_{ij} is the metric tensor and T_{ij} is the energy-momentum tensor for a cloud string respectively.

Thus the energy-momentum tensor for a cloud string is given by

$$T_{ij} = \rho \nu_i \nu_j - \lambda x_i x^j \tag{3.3}$$

where ν_i and x_i satisfy the conditions

$$\nu^i \nu_i = -x^i x_i = -1, \nu^i x_i = 0 \tag{3.4}$$

Here ρ is the rest energy density for a cloud of strings with particles attached to them. $\rho = \rho_p + \lambda$, ρ_p being the rest energy density of particles attached to the strings and λ the tension density of the strings. Here p and ρ are a function of cosmic time t only. x_i

is a unit space-like vector instead of the direction of strings so that $x^2 = x^3 = x^4 = 0$ and $x^1 \neq 0$.

The energy-momentum tensor T_{ij} in co-moving coordinates for could string is given by

$$T_0^0 = \rho, T_1^1 = \lambda, T_2^2 = T_3^3 = T_4^4 = 0 \tag{3.5}$$

The field eqn. (3.2) for the line-element (3.1) with the help of (3.3) -(3.5) can be written explicitly as

$$3\frac{\dot{A}^2}{A^2} + 3\frac{\dot{A}\dot{B}}{AB} = \rho \tag{3.6}$$

$$2\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + 2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}^2}{A^2} = \lambda \tag{3.7}$$

$$2\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + 2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}^2}{A^2} = 0 \tag{3.8}$$

$$3\frac{\ddot{A}}{A} + 3\frac{\dot{A}^2}{A^2} = 0 \tag{3.9}$$

An over dot indicates a derivative with respect to cosmic time t.

The spatial volume for the model (3.1) is given by

$$V = a^4 = A^3 B \tag{3.10}$$

3.3 Solution of the Field Equations

To solve the system of equations we consider deceleration parameter (q) as a linear function of Hubble's parameter (Tiwari et al. (2015); Tiwari et al. (2018); Sharma et al. (2019)).

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = \beta H + \alpha \tag{3.11}$$

Here α and β are arbitrary constants.

For $\alpha = -1$ in eqn. (3.11)

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -1 + \beta H$$

which yields the following differential equation

$$\frac{a\ddot{a}}{\dot{a}^2} + \beta\frac{\dot{a}}{a} - 1 = 0 \tag{3.12}$$

After integrating eqn. (3.12) we get

$$a(t) = e^{\frac{1}{\beta}\sqrt{2\beta t + c}} \tag{3.13}$$

where c is an integrating constant.

Collins et al. (1980) have explored that for a spatially homogeneous metric, a large class of solutions that can satisfy the condition $\frac{\sigma}{\theta}$ is constant, where θ is the expansion scalar in the model. So we assume the shear scalar σ is proportional to the expansion scalar θ. This gives the relation between scale factor A and B as,

$$A = B^n \tag{3.14}$$

where n is constant and $n \neq 1$.

From eqns. (3.10) , (3.13) and (3.14), the metric component are

$$A(t) = e^{\frac{4n}{\beta(3n+1)}\sqrt{2\beta t + c}} \tag{3.15}$$

and

$$B(t) = e^{\frac{4}{\beta(3n+1)}\sqrt{2\beta t + c}} \tag{3.16}$$

Therefore the metric (3.1) reduces to

$$ds^2 = dt^2 - e^{\frac{4n}{\beta(3n+1)}\sqrt{2\beta t + c}}(dx^2 + dy^2 + dz^2) - e^{\frac{4}{\beta(3n+1)}\sqrt{2\beta t + c}}d\phi^2 \tag{3.17}$$

The eqn. (3.17) represents 5D Kaluza-Klein cosmological model with variable deceleration parameter.

3.4 Physical Parameters of the Model and its Behaviour

From the cosmological model (3.17), the directional Hubble's parameters H_x, H_y, H_z and H_ϕ, the physical quantities such as Hubble's parameter H, spatial volume V, anisotropy parameter Δ, expansion scalar θ, shear scalar σ^2, energy density ρ, particles density ρ_p and tension density of the string λ are obtained as follows

The directional Hubble's parameters H_x, H_y, H_z and H_ϕ are

$$H_x = H_y = H_z = \frac{4n}{(3n+1)\sqrt{2\beta t + c}}$$

and

$$H_\phi = \frac{4}{(3n+1)\sqrt{2\beta t + c}}$$

For Kaluza-Klein space-time, the Hubble's parameter(H) is given by

$$H = \frac{1}{\sqrt{2\beta t + c}} \tag{3.18}$$

The spatial volume(V) is given by

$$V = e^{\frac{4}{\beta}\sqrt{2\beta t + c}} \tag{3.19}$$

The expansion of anisotropic parameter (Δ) is given by

$$\Delta = \frac{3(n-1)^2}{(3n+1)^2} = \text{ constant } (\neq 0 \text{ where } n \neq 1) \tag{3.20}$$

The expansion scalar (θ) is given by

$$\theta = \frac{4}{\sqrt{2\beta t + c}} \tag{3.21}$$

The shear scalar (σ^2) is given by

$$\sigma^2 = \frac{6(n-1)^2}{(3n+1)^2(2\beta t + c)} \tag{3.22}$$

From eqns. (3.21) and (3.22) we get

$$\lim_{t\to\infty} \frac{\sigma^2}{\theta^2} = \frac{3(n-1)^2}{8(3n+1)^2} = \text{ constant } (\neq 0 \text{ where } n \neq 1) \tag{3.23}$$

The energy density ρ is given by

$$\rho = \frac{48n(n+1)}{(3n+1)^2(2\beta t + c)} \tag{3.24}$$

The tension density λ for the string is given by

$$\lambda = \frac{16(3n^2+2n+1)}{(3n+1)^2(2\beta t + c)} - \frac{4\beta(2n+1)}{(3n+1)(2\beta t + c)^{\frac{3}{2}}} \tag{3.25}$$

The particle density ρ_p attached to the string is given by

$$\rho_p = \frac{16(n-1)}{(3n+1)^2(2\beta t + c)} + \frac{4\beta(2n+1)}{(3n+1)(2\beta t + c)^{\frac{3}{2}}} \tag{3.26}$$

The deceleration parameter (q) is given by

$$q = -1 + \frac{\beta}{\sqrt{2\beta t + c}} \tag{3.27}$$

Now we discuss the physical and geometrical behaviour of the cosmological parameters. We have plotted the various figures with respect to cosmic time t by taking $n = \beta = 1, n = \beta = 2, n = \beta = 3$ and $c = 1$ for the model are shown below:

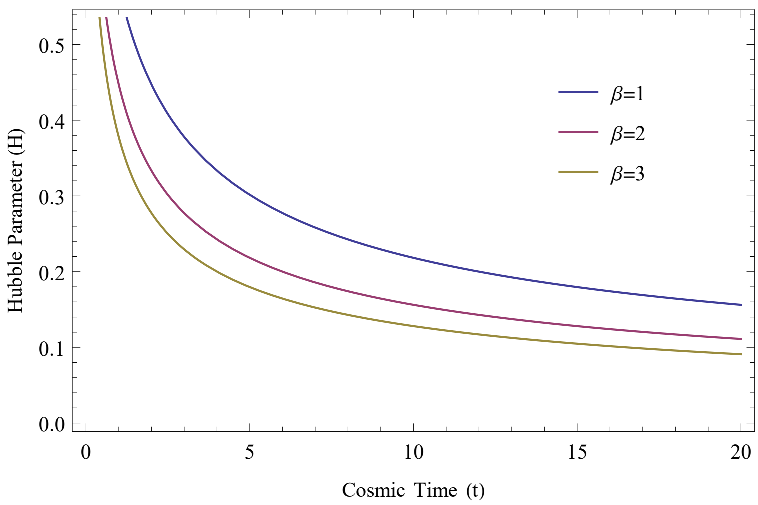

Figure 3.1: The plot of H versus cosmic time t.

From eqns. (3.18) and (3.21), it is observed that the Hubble's parameter H and the expansion scalar θ are both positive and decreasing functions of cosmic time t. For both H and θ tend to infinity as $t \to 0$ and tends to a finite value, when $t \to \infty$ are shown in

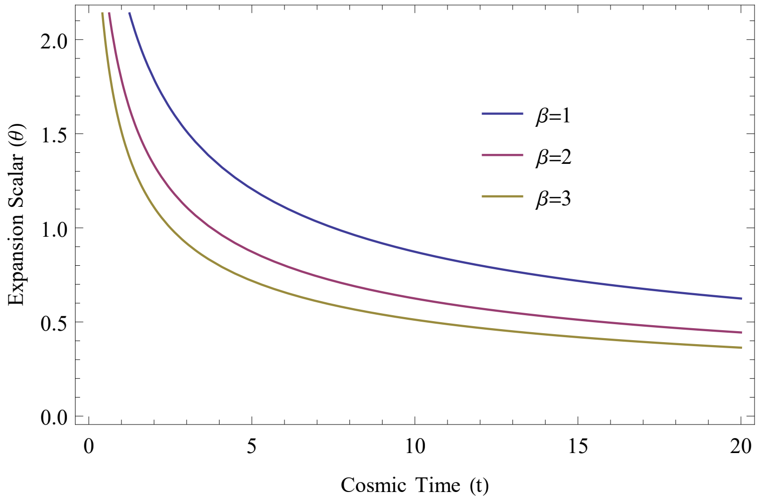

Figure 3.2: The plot of θ versus cosmic time t

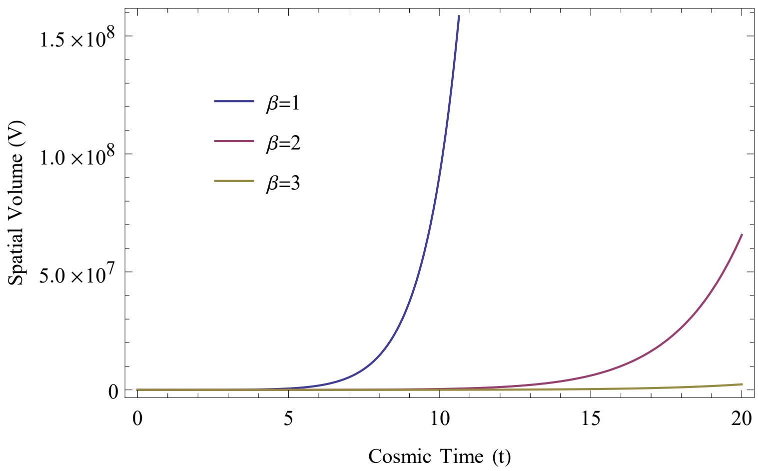

Figure 3.3: The plot of V versus cosmic time t.

fig.-3.1 and fig.-3.2. The proposed model universe has a point type initial singularity (MacCallum (1971)). We have also noticed that $\frac{dH}{dt}$ is negative, which suggests that the universe is in accelerated phase of expansion.

Eqn. (3.19) shows that at $t = 0$ the spatial volume V is finite and thereafter increases continuously when cosmic time t is increasing which shows that the model universe is expanding with the evolution of time. The nature of fluctuations in V versus cosmic time t is seen in fig. 3.3.

From eqn. (3.27), we observed that the deceleration parameter $q < 0$ for $t > \frac{\beta^2 - c}{2\beta}$, which suggests that the model universe is in a accelerated phase of expansion, which are

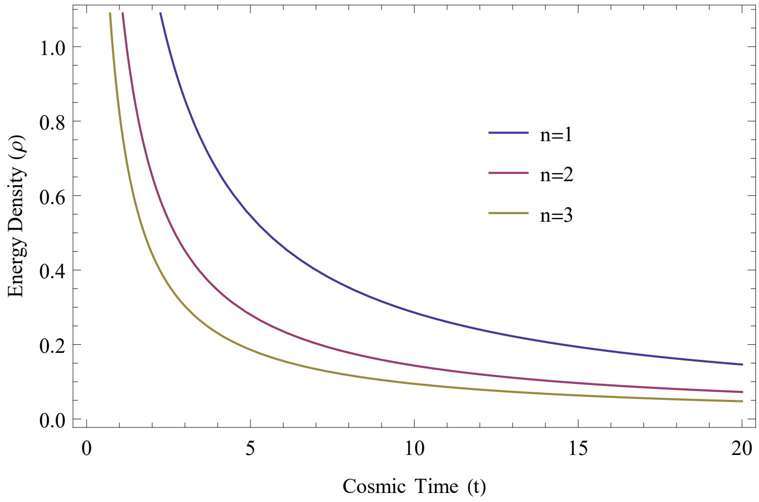

Figure 3.4: The plot of ρ versus cosmic time t.

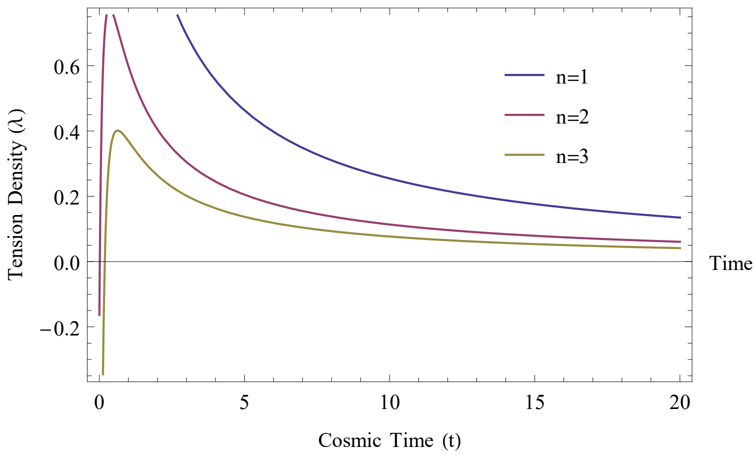

Figure 3.5: The plot of λ versus cosmic time t

in agrees with present day's observations (Riess et al. (1998); Garnavich et al. (1998); Schmidt et al. (1998); Perlmutter et al. (1999)).

The expansion of anisotropic parameter $\Delta \neq 0$ (constant) for $n \neq 1$ and $\Delta = 0$ for $n = 1$. We also observed from eqn. (3.23) that $\lim_{t\to\infty} \frac{\sigma^2}{\theta^2} \neq 0$ (constant), so, the model universe is anisotropic one. Though anisotropy is included into the system, it does not conflict with current evidence that the universe is isotropy. This is due to the fact that during the process of evolution, the initial anisotropy gradually vanishes after some duration and approaches to the isotropy and eventually it evolves into a FRW universe as proposed by Jacobs (Jacobs (1968)).

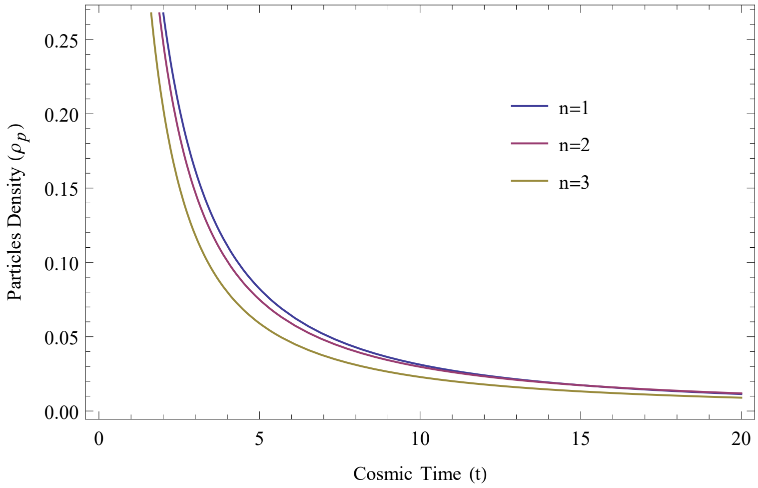

Figure 3.6: The plot of the ρ_p versus cosmic time t.

From fig-3.4 corresponding to the eqn. (3.24), it is seen that the rest energy density ρ is decreases when time t is increases and initially $\rho \to \infty$ when $t \to 0$, thus has a point type initial singularity (MacCallum (1971)). Also from this figure, it is observed that the rest energy density is positive and satisfies the energy condition $\rho \geq 0$ for all $n \geq -1$, which shows that the model universe is in a state of accelerating expansion.

It is seen that from fig.-3.5 and fig.-3.6, both the string tension density λ and particle density ρ_p are positive, decreasing function of cosmic time t. Initially both the string tension density λ and particle density ρ_p tends to infinity when t tends to zero and become zero as $t \to \infty$ which suggests that the universe began with big bang and as time progresses, both the string tension density λ and particle density ρ_p decreases with the expansion of the universe.