Chapter 4

Two-Fluid Scenario for Dark Energy Cosmological Model in Five Dimensional Kaluza-Klein Space-Time

4.1 Introduction

In 1998 and the years after, numerous cosmological observations such as (Perlmutter et al. (1998); Riess et al. (1998); Perlmutter et al. (1999); Bennett et al. (2003); Spergel et al. (2003); Riess et al. (2004)) that the universe is not only expanding but also the acceleration of the expansion from the big-bang till today. DE is the phenomenon responsible for the accelerating expansion of the universe. Nowadays, cosmologists have developed many theoretical models to explain the existence of DE.

The evolution of the DE parameter under two-fluid scenario by considering different form of deceleration parameter has been investigated many prominent authors (Setare et al. (2009); Jamil et al. (2010); Amirhashchi et al. (2011a); Amirhashchi et al. (2011b); Pradhan et al. (2011); Singh and Chaubey (2012); Singh and Chaubey (2013); Saha et al. (2012); Reddy and Kumar (2013); Samanta and Debata (2013); Pradhan (2014); Amirhashchi et al. (2014); Rao et al. (2016); Rao and Sireesha (2017); Tripathy et al. (2017); Tiwari et al. (2017); Tiwari et al. (2018); Tiwari et al. (2018); Goswami et al. (2020); Mishra et al. (2021)). Recently Kumar et al. (2022) investigated two-fluids cosmological models with matter and radiating source in $(2+1)-$ dimensional Saez-

Ballester scalar-tensor theory of gravitation. Hatkar et al. (2022) and Trivedi and Bhabor (2022) has explored the features of Bianchi type-I cosmological universe under two-fluid scenario within the framework of different theories of gravitation.

In this chapter, we have investigated the 5D Kaluza-Klein homogeneous and isotropic dark energy cosmological model under two-fluid scenario with special form of deceleration parameter.

4.2 The metric and Field Equations

The FRW type homogeneous and isotropic 5D Kaluza-Klein space-time has been considered and it is as follows

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) + (1-kr^2)d\psi^2\right] \tag{4.1}$$

where $a(t)$ is a scale factor considered to be a function of cosmic time t and $k = -1, 0, +1$ is the curvature parameter for open, flat and closed universe respectively.

The Einstein's field eqn. (with $8\pi G = 1$ and $c = 1$) can be written as

$$R_{ij} - \frac{1}{2}g_{ij}R = -T_{ij} \tag{4.2}$$

where T_{ij} is the two fluid energy momentum tensor consisting of dark fluid and barotropic fluid.

The energy momentum tensor for two fluid is given by

$$T_{ij} = T_{ij}^m + T_{ij}^D \tag{4.3}$$

where

$$T_{ij}^m = (\rho_m + p_m)]\mu_i\mu_j - p_m g_{ij} \tag{4.4}$$

$$T_{ij}^D = (\rho_D + p_D)\mu_i\mu_j - p_D g_{ij} \tag{4.5}$$

here ρ_m and p_m are the energy density and pressure of the perfect fluid and ρ_D and p_D are the energy density and pressure of the DE component.

The Einstein field eqn (4.2) with (4.3) for the metric (4.1) we may write

$$6\left(\frac{\dot{a}^2}{a^2}+\frac{k}{a^2}\right)=(\rho_m+\rho_D) \tag{4.6}$$

and

$$3\frac{\ddot{a}}{a}+3\left(\frac{\dot{a}^2}{a^2}+\frac{k}{a^2}\right)=-(p_m+p_D) \tag{4.7}$$

Here the over dot indicate a derivatives with respect to cosmic time t.

The energy conservation equation $T^{ij}_{;j}=0$, which yields

$$\dot{\rho}+4\frac{\dot{a}}{a}(\rho+p)=0 \tag{4.8}$$

where $p=p_m+p_D$ and $\rho=\rho_m+\rho_D$.

The EoS parameter (ω) which is considered as an important quantity in describing the dynamics of the universe, is given by

$$p_m=(\omega_m-1)\rho_m \tag{4.9}$$

$$p_D=(\omega_D-1)\rho_D \tag{4.10}$$

Here, we consider two cases:

Case-I: In which the two fluids do not-interact with each other and

Case-II: When the two fluids interact each other.

4.3 Case-I: Non-interacting Two Fluid Model

In this section, the fluids do not interact with each other. The conservation equation for the dark and barotropic fluid separatly as

$$\dot{\rho_m}+4\frac{\dot{a}}{a}(\rho_m+p_m)=0 \tag{4.11}$$

$$\dot{\rho_D}+4\frac{\dot{a}}{a}(\rho_D+p_D)=0 \tag{4.12}$$

Integrating (4.11) we obtain

$$\rho_m = \rho_0 a^{-4\omega_m} \tag{4.13}$$

where ρ_0 is an integrating constant.

Now using (4.13) in (4.6) and (4.7) we obtain ρ_D and p_D in terms of scale factor $a(t)$.

$$\rho_D = 6\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) - \rho_0 a^{-4\omega_m} \tag{4.14}$$

$$p_D = -3\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) - \rho_0(\omega_m - 1)a^{-4\omega_m} \tag{4.15}$$

Now we assume the special form of deceleration parameter (Banerjee and Das (2005); Singha and Debnath (2009); Sahoo et al. (2018)) is given by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -1 + \frac{\alpha}{1 + a^\alpha} \tag{4.16}$$

where α is an arbitrary constant.

After integrating (4.16) we get

$$a(t) = (e^{m\alpha t} - 1)^{\frac{1}{\alpha}} \tag{4.17}$$

where m is an integration constant.

From eqns. (4.16) and (4.17), the deceleration parameter q as use

$$q = \frac{-a\ddot{a}}{\dot{a}^2} = -1 + \frac{\alpha}{e^{m\alpha t}} \tag{4.18}$$

By using the scale factor $a(t)$ in (4.14) and (4.15) we obtain

$$\rho_D = 6[\frac{m^2 e^{2m\alpha t}}{(e^{m\alpha t} - 1)^2} + \frac{k}{(e^{m\alpha t} - 1)^{\frac{2}{\alpha}}}] - \rho_0(e^{m\alpha t} - 1)^{\frac{-4\omega_m}{\alpha}} \tag{4.19}$$

and

$$p_D = -3[\frac{m^2 e^{m\alpha t}(2e^{m\alpha t} - \alpha + 1)}{(e^{m\alpha t} - 1)^2} + \frac{k}{(e^{m\alpha t} - 1)^{\frac{2}{\alpha}}}] - \rho_0(\omega_m - 1)(e^{m\alpha t} - 1)^{\frac{-4\omega_m}{\alpha}} \tag{4.20}$$

By using (4.19) and (4.20) in (4.10) we obtain

$$\omega_D = -\left[\frac{\frac{3m^2e^{m\alpha t}(2e^{m\alpha t}-\alpha+1)}{(e^{m\alpha t}-1)^2} + \frac{3k}{(e^{m\alpha t}-1)^{\frac{2}{\alpha}}} + \rho_0(\omega_m - 1)(e^{m\alpha t}-1)^{\frac{-4\omega_m}{\alpha}}}{\frac{6m^2e^{2m\alpha t}}{(e^{m\alpha t}-1)^2} + \frac{6k}{(e^{m\alpha t}-1)^{\frac{2}{\alpha}}} - \rho_0(e^{m\alpha t}-1)^{\frac{-4\omega_m}{\alpha}}}\right] + 1 \tag{4.21}$$

The expressions for the matter energy density Ω_m and DE density Ω_D are given by

$$\Omega_m = \frac{\rho_m}{6H^2} = \frac{\rho_0(e^{m\alpha t}-1)^{\frac{-4\omega_m}{\alpha}+2}}{6m^2e^{2m\alpha t}} \tag{4.22}$$

and

$$\Omega_D = \frac{\rho_D}{6H^2} = 1 + \frac{k(e^{m\alpha t}-1)^2}{6m^2e^{2m\alpha t}(e^{m\alpha t}-1)^{\frac{2}{\alpha}}} - \frac{\rho_0(e^{m\alpha t}-1)^{\frac{-4\omega_m}{\alpha}+2}}{6m^2e^{2m\alpha t}} \tag{4.23}$$

From eqns. (4.22) and (4.23) we obtain

$$\Omega = \Omega_m + \Omega_D = 1 + \frac{k(e^{m\alpha t}-1)^2}{6m^2e^{2m\alpha t}(e^{m\alpha t}-1)^{\frac{2}{\alpha}}} \tag{4.24}$$

4.4 Case-II: Interacting Two Fluid Model

In this section we consider the interaction between dark fluid and barotropic fluid. The conservation equation for the dark and barotropic fluid are given by

$$\dot{\rho_m} + 4\frac{\dot{a}}{a}(\rho_m + p_m) = Q \tag{4.25}$$

$$\dot{\rho_D} + 4\frac{\dot{a}}{a}(\rho_D + p_D) = -Q \tag{4.26}$$

where the quantity Q represents the interaction between the matter and DE component. We consider $Q > 0$, as it shows that the energy transferred from DE to dark matter. Following Amendola et al. (2007); Guo et al. (2007), we consider

$$Q = 4H\sigma\rho_m \tag{4.27}$$

where σ is an coupling constant.

Using (4.27) in (4.25) and after integrating we obtain

$$\rho_m = \rho_0 a^{-4(\omega_m-\sigma)} \tag{4.28}$$

where ρ_0 is an integrating constant.

Now using (4.28) in (4.14) and (4.15) we obtain ρ_D and p_D in terms of scale factor $a(t)$.

$$\rho_D = 6\left(\frac{\dot{a}^2}{a^2}+\frac{k}{a^2}\right) - \rho_0 a^{-4(\omega_m-\sigma)} \tag{4.29}$$

$$p_D = -3\left(\frac{\dot{a}}{a}+\frac{\dot{a}^2}{a^2}+\frac{k}{a^2}\right) - \rho_0(\omega_m-1)a^{-4(\omega_m-\sigma)} \tag{4.30}$$

Putting the value of $a(t)$ from (4.17) in eqns. (4.29) and (4.30) we obtain

$$\rho_D = 6[\frac{m^2e^{2m\alpha t}}{(e^{m\alpha t}-1)^2}+\frac{k}{(e^{m\alpha t}-1)^{\frac{2}{\alpha}}}] - \rho_0(e^{m\alpha t}-1)^{\frac{-4(\omega_m-\sigma)}{\alpha}} \tag{4.31}$$

and

$$p_D = -3[\frac{m^2e^{m\alpha t}(2e^{m\alpha t}-\alpha+1)}{(e^{m\alpha t}-1)^2}+\frac{k}{(e^{m\alpha t}-1)^{\frac{2}{\alpha}}}] - \rho_0(\omega_m-1)(e^{m\alpha t}-1)^{\frac{-4(\omega_m-\sigma)}{\alpha}} \tag{4.32}$$

Using (4.31) and (4.32) in (4.10) we obtain

$$\omega_D = -\left[\frac{\frac{3m^2e^{m\alpha t}(2e^{m\alpha t}-\alpha+1)}{(e^{m\alpha t}-1)^2}+\frac{3k}{(e^{m\alpha t}-1)^{\frac{2}{\alpha}}}] + \rho_0(\omega_m-1)(e^{m\alpha t}-1)^{\frac{-4(\omega_m-\sigma)}{\alpha}}}{\frac{6m^2e^{2m\alpha t}}{(e^{m\alpha t}-1)^2}+\frac{6k}{(e^{m\alpha t}-1)^{\frac{2}{\alpha}}} - \rho_0(e^{m\alpha t}-1)^{\frac{-4(\omega_m-\sigma)}{\alpha}}}\right]+1 \tag{4.33}$$

The expressions for the matter energy density Ω_m and DE density Ω_D are given by

$$\Omega_m = \frac{\rho_m}{6H^2} = \frac{\rho_0(e^{m\alpha t}-1)^{\frac{-4(\omega_m-\sigma)}{\alpha}+2}}{6m^2e^{2m\alpha t}} \tag{4.34}$$

and

$$\Omega_D = \frac{\rho_D}{6H^2} = 1+\frac{k(e^{m\alpha t}-1)^2}{6m^2e^{2m\alpha t}(e^{m\alpha t}-1)^{\frac{2}{\alpha}}} - \frac{\rho_0(e^{m\alpha t}-1)^{\frac{-4(\omega_m-\sigma)}{\alpha}+2}}{6m^2e^{2m\alpha t}} \tag{4.35}$$

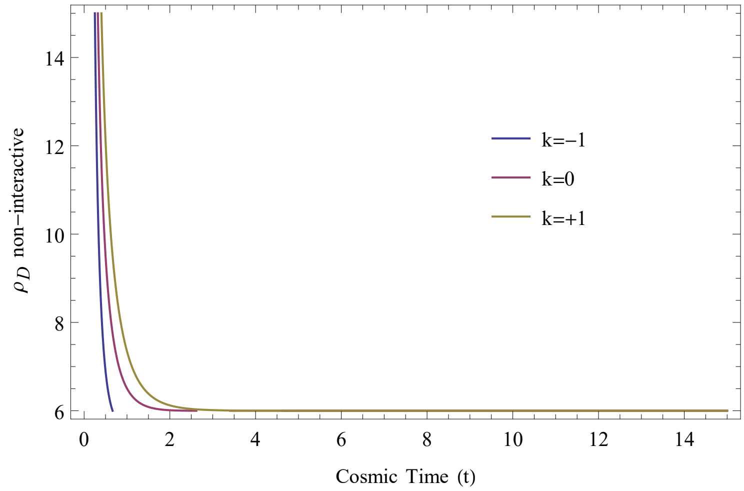

Figure 4.1: The plot of ρ_D versus cosmic time t with $m = 1$,$\rho_0 = 1$,$\alpha = 3$,$\omega_m = 0.5$ and $k = -1, 0, +1$ for non-interactive case.

From eqns. (4.34) and (4.35) we obtain the total energy density parameter Ω as

$$\Omega = \Omega_m + \Omega_D = 1 + \frac{k(e^{m\alpha t} - 1)^2}{6m^2 e^{2m\alpha t}(e^{m\alpha t} - 1)^{\frac{2}{\alpha}}} \tag{4.36}$$

For the derived model, the jerk parameter (j) can be written as

$$j(t) = 1 - \frac{3\alpha}{e^{m\alpha t}} + \frac{\alpha^2(e^{m\alpha t} + 3)}{e^{2m\alpha t}} \tag{4.37}$$

4.5 Physical Interpretation of the Solutions:

We have plotted the various figures using these calculations for suitable values of the constants. We also used all the three values of $k = -1, 0, 1$ while plotting the figures.

Figs. 4.3 and 4.4 represent the variation of pressure for DE for both cases. We observed that at $t \to 0$ the pressure p_D is negative for open ($k = -1$) and flat ($k = 0$) universe and positive for closed ($k = 1$) universe. Finally, pressure p_D is zero all the three open, closed and flat universe for late time cosmic evolution.

The behavior of EoS for DE with respect to cosmic time t is shown in figs. 4.5 and 4.6 which correspond to the eqns. (4.20) and (4.33) for both non-interacting and interacting cases respectively. We observed that from both the figures the EoS parameter ω_D is a decreasing function of time t. At $t \to \infty$, the EoS parameter ω_D tends to zero. So, the model indicating matter dominated era of the universe .

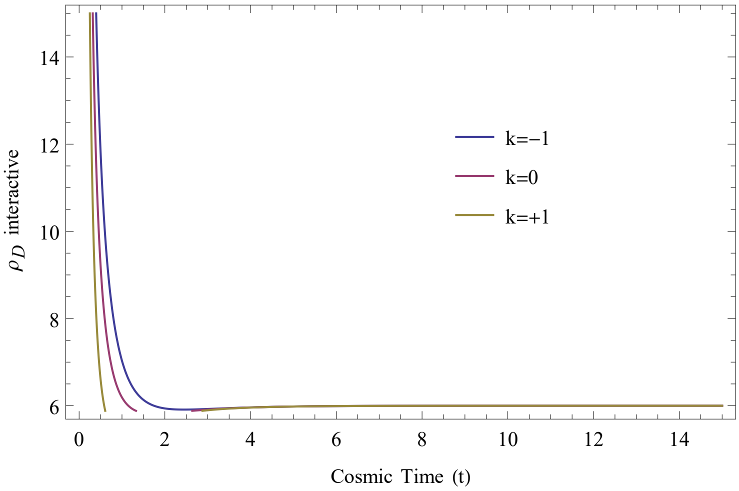

Figure 4.2: The plot of ρ_D versus cosmic time t with $m = 1$,$\rho_0 = 1$,$\alpha = 3$,$\omega_m = 0.5$ and $k = -1, 0, +1$ for interactive case.

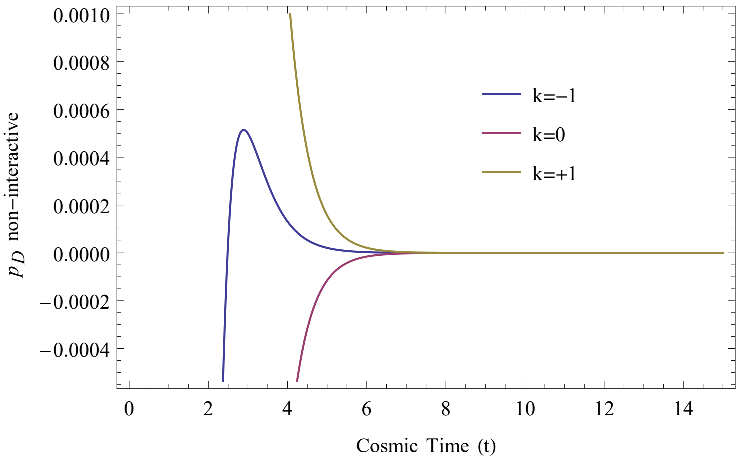

Figure 4.3: The plot of p_D versus cosmic time t with $m = 1$,$\rho_0 = 1$,$\alpha = 3$,$\omega_m = 0.5$ and $k = -1, 0, +1$ for non-interactive case.

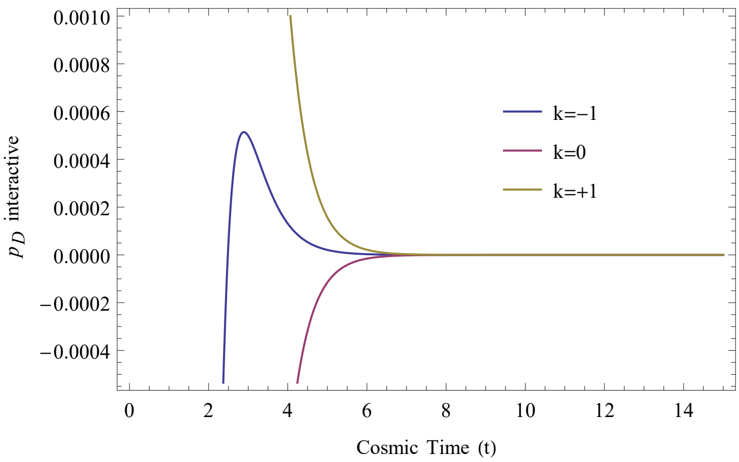

Figure 4.4: The plot of p_D versus cosmic time t with $m = 1$,$\rho_0 = 1$,$\alpha = 3$,$\omega_m = 0.5$ and $k = -1, 0, +1$ for interactive case.

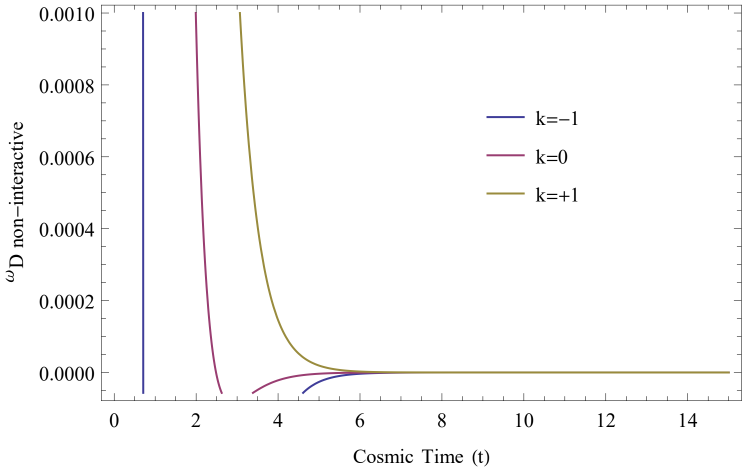

Figure 4.5: The plot of ω_D versus cosmic time t with $m = 1$,$\rho_0 = 1$,$\alpha = 3$,$\omega_m = 0.5$ and $k = -1, 0, +1$ for non-interactive case.

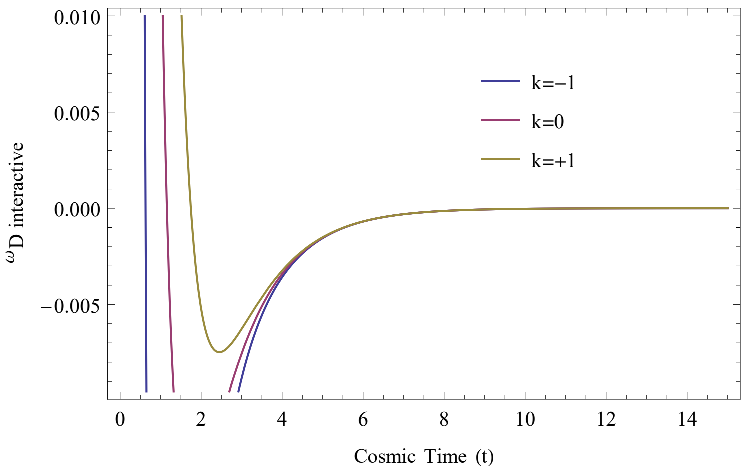

Figure 4.6: The plot of ω_D versus cosmic time t with $m = 1$, $\rho_0 = 1$, $\alpha = 3$, $\omega_m = 0.5$ and $k = -1, 0, +1$ for interactive case.

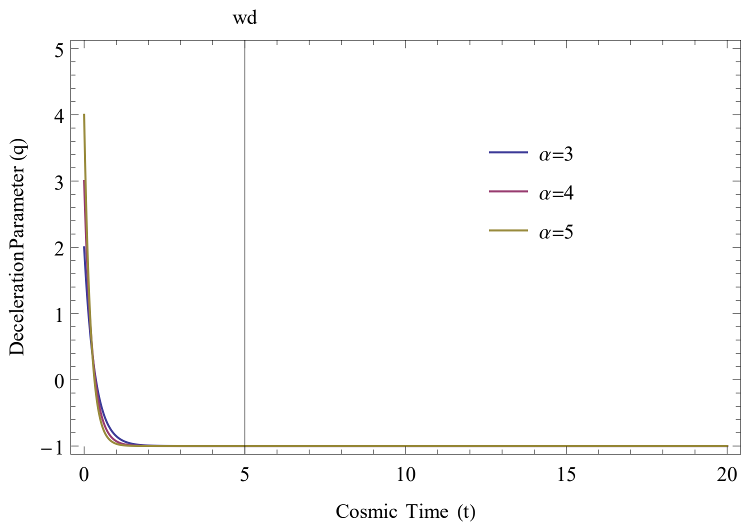

Figure 4.7: The plot of q versus cosmic time t with $m = 1$,$\alpha = 3, 4, 5$.

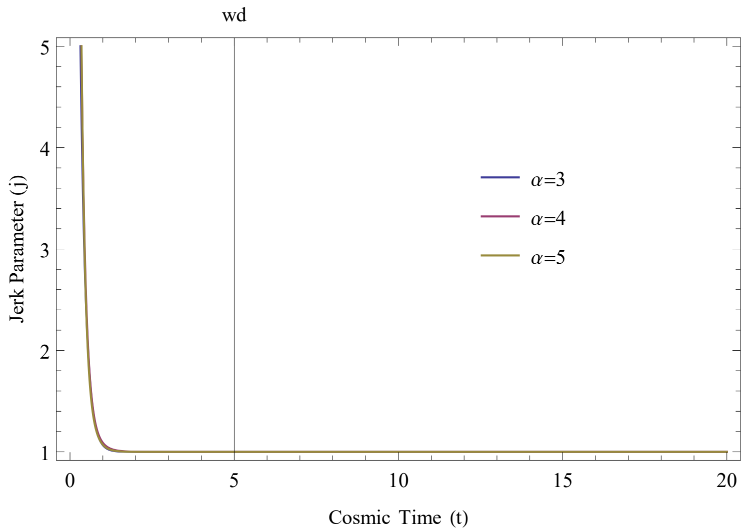

Figure 4.8: The plot of j versus cosmic time t with $m = 1$,$\alpha = 3, 4, 5$.

From eqns. (4.23) and (4.36) the total energy density parameter Ω for both non-interacting and interacting cases are same. From the right hand side of these two equations it is clear that for $k = 0$ (i.e for flat universe), $\Omega = 1$ and $k = 1$ (i.e for closed universe), $\Omega > 1$ and $k = -1$ (i.e for open universe), $\Omega < 1$. We also observed that, at late time, Ω approaches to 1 for all the three values of k, which is shows that the universe will acquire a flat structure. These results are fully consistent with the present day's observations.

Initially, at $t \to 0$ the deceleration parameter q is positive and then, it decreases with cosmic time t is increase and at $t \to \infty$, deceleration parameter $q \to -1$, which shows that the proposed model universe has a transition from decelerating phase to accelerating phase of expansion.

A decelerate phase to accelerate phase transition of the universe occurs for models with a positive value of jerk parameter and negative value of deceleration parameter. In figs. 4.7 and 4.8 we observed that deceleration parameter q is negative and jerk parameter j is positive so that we do have a transition of the model from decelerated to accelerated phase at late time cosmic evolution.