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List of Publications and Presentations

List of Publications

1. Basumatary B., Khaklary, J.K., & Said B. (2022). On NeutroBitopological Spaces, *International Journal of Neutrosophic Science*,**18**(2), 254-261.
2. Basumatary B., Khaklary J.K., Wary N., & Smarandache F. (2022). On Neutro-Topological Spaces and their Properties.*Theory and Applications of NeutroAlgebras as Generalisations of Classical Algebras*. 180-201. *IGI Publications*.
3. Basumatary B., & Khaklary J.K. (2022). A Study on the Properties of Anti-Topological Space. 16-27. *Neutrosophic Algebraic Structures and Their Applications. NSIA Publications.*
4. Basumatary B., & Khaklary J.K. (2024). A Study on Continuity functions in neutro-topological spaces, *Neutrosophic Sets and Systems*, **78**, 341-352.

Papers presented in Conference

1. National Conference on "Science & Technology for Sustainable Development (STSD-2022)" on 9th–10th September, 2022 jointly organised by Science College Kokrajhar & Vijnana Bharati, NESM in collaboration with IASST, Guwahati and NECTAR, Shillong.
2. National Conference on "Advances in Mathematical Sciences" 22nd-23rd December, 2022 organised by Department of Mathematics, Gauhati University.

Chapter 11
On Neutro-Topological Spaces and Their Properties

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ABSTRACT

There is a lot of ambiguous information in the real world that crisp values can't manage. The fuzzy set theory was proposed by Zadeh. It is an age-old and excellent tool for dealing with uncertain information. As a result, intuitionistic fuzzy set theory was suggested. However, these theories are incapable of dealing with all forms of uncertainty, such as indeterminate and inconsistent data in various decision-making situations. To address this shortfall, Smarandache proposed the neutrosophic set theory by introducing a degree of indeterminacy as an independent component. In this current decade, neutrosophic environments are mainly interested by different fields of researchers. Recently, Smarandache introduced theNeutroAlgebra and AntiAlgebras. NeutroAlgebras and AntiAlgebras represent a new research subject that is based on real-world examples. In this chapter, some properties of NeutroTopological space are introduced and studied with examples. Several definitions of NeutroInterior, NeutroClosure, and NeutroBoundary are defined, and the authors also studied its properties with examples.

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INTRODUCTION

The concept of a fuzzy set was first developed by Zadeh (1965), where the concept of the membership function is defined and discussed the concept of uncertainty using a fuzzy set. Atanassov (1986) proposed the intuitionistic fuzzy set by generalizing the concept of fuzzy sets and introducing the degree of non-membership as a component. Chang (1968) was the first to introduce fuzzy topology, while Coker (1997) defined intuitionistic fuzzy topological space. Salama et al. (2014) investigated topology using neutrosophic sets. Kelly (1963) introduced the concept of bitopological space. The concept of fuzzy bitopological space was investigated by Kandil et al. (1995). Florentin Smarandache (1998) defined the idea of neutrosophic logic and the concept of neutrosophic set. After that, the concepts of the neutrosophic set have been applied in many branches of sciences and technology. The concept of neutrosophic topological space was introduced by Salama and Alblowi (2012). Devi *et al*. (2017) discussed separation axioms in ordered neutrosophic bitopological space. Mwchahary et al. (2020) did their work in neutrosophic bitopological space. After defining the neutrosophic group, Sumathi et al. (2016) defined the concept of the topological group structure of the neutrosophic set and also, Sumathi and Arockiarani (2015) studied the fuzzy neutrosophic group.

In recent years, there has been a surge in academic interest in neutrosophic set theory. The concept of neutro-structures and anti-structures was first defined by Florentin Smarandache (2019, 2020). Şahin et al. (2021) discussed the idea of neutro-topological space and anti-topological space. Smarandache (2020) studied NeutroAlgebra as a generalization of partial algebra. Agboola (2020) investigated the idea of NeutroRings, NeutroGroups, and finite NeutroGroups of type-NG. Smarandache (2020) proposed the generalizations and alternatives of Classical Algebraic Structures to NeutroAlgebraic Structures and AntiAlgebraic Structures. Al-Tahan et al. (2021) studied the NeutroOrderedAlgebra, NeutroHyper structures, and their properties.

Blizard (1989) investigated the concept of multiset theory and neutrosophic multi groups and applications were studied by Bakbak (2019). Basumatary et al. (2020) studied on interval-valued triangular neutrosophic linear programming problem also Basumatary et al. (2021) investigated some properties of the neutrosophic multi topological group. Yager (1986) introduced the concept of fuzzy multiset (fuzzy bag) to generalize the fuzzy set, and Miyamoto (2001) studied Fuzzy Multisets and Their Generalizations. A fuzzy multiset element can appear more than once, with the same or different membership values. Onasanya et al. (2018) investigated the Algebraic properties of alpha-level subsets topology of a fuzzy subset, and the authors Onasanya et al. (2019) and Al Tahan et al. (2020) studied fuzzy multi-polygroups. Al Tahan et al. (2019) studied the fuzzy multi-Hv-ideals of Hv-rings fuzzy multi-Hv-ideals and

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On NeutroBitopological Space

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Abstract

The current study shows the study of NeutroBitopological Space. In this work, the properties of NeutroBitopological Space are discussed. It is seen that many properties do not coincide with the properties of general Bitopological space. The terms NeutroInterior, NeutroClosure, and NeutroBoundary are defined with examples also their properties are observed.

Keywords: NeutroInterior, NeutroClosure, NeutroBoundary, NeutroBitopological Space.

1.Introduction

Smarandache [1, 2] proposed the neutrosophic set (NS), and after that many researchers applied it in science & technology. In recent years, there has been a surge in academic interest in neutrosophic set theory. Florentin Smarandache first defined the idea of neutro-structures and anti-structures [3, 4]. Neutrosophication of an axiom on a given set X means dividing the set X into three regions, one in which the axiom is true (we call this the degree of truth T of the axiom), one in which the axiom is indeterminate (we call this the degree of indeterminacy I of the axiom), and one in which the axiom is false (we call this the degree of indeterminacy I of the axiom (we say the degree of falsehood F of the axiom). On, the other hand Antisophication of an axiom on a given set X means to have the axiom false on the whole set X. Without using neutrosophic sets and neutrosophic numbers, the structure of neutrosophic logic has been transferred to the structure of classical algebras. MemetŞahin et al. [8] studied NeutroTopological Space (NTS) and Anti-Topological Space. Smarandache [7] studied neutroAlgebra as a generalization of partial algebra. Many researchers [11-15] studied neutroAlgebra. Basumataryet al. [21] studied some properties of NTS.

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In this paper, NeutroBitopological Space is studied based on NeutroInterior, NeutroClosure, and NeutroBoundary, also its properties are learned with examples.

2. Preliminaires

Definition 2.1: [7]

TheNeutro-sophication of the Law

(i) Let X be a non-empty set and $*$ be binary operation. For some elements $(a,b) \in (X,X)$, $(a*b) \in X$ (degree of well defined (T)) and for other elements $(x,y),(p,q) \in (X,X)$; $[x*y$ is indeterminate (degree of indeterminacy (I)), or $p*q \notin X$ (degree of outer-defined (F))], where (T,I,F) is different from (1,0,0) that represents the Classical Law, and from (0,0,1) that represents the AntiLaw.

(ii) In NeutroAlgebra, the classical well-defined for $*$ binary operation is divided into three regions: degree of well-defined (T), degree of indeterminacy (I) and degree of outer-defined (F) similar to neutrosophic set and neutrosophic logic

Definition 2.2: [8]

Let X be a non-empty set, $\Im$ be a collection of subsets of X. If at least one of the following conditions {i, ii, iii} is satisfied, then $\Im$ is called a NeutroTopology on X and (X, $\Im$) is called a **NTS**.

(i) $[\emptyset \in \Im, X \notin \Im$ or $X \in \Im, \emptyset \notin \Im]$ or $[\emptyset, X \in_1 \Im]$

(ii) For at least n elements $p_1, p_2, \dots, p_n \in \Im$, $\cap_{i=1}^{n} p_i \in \Im$ and for at least n elements $q_1, q_2, \dots, q_n \in \Im$, $r_1, r_2, \dots, r_n \in \Im$; $[\cap_{i=1}^{n} q_i \notin \Im$ or $\cap_{i=1}^{n} r_i \in_1 \Im]$. Where n is finite.

(iii) For at least n elements $p_1, p_2, \dots, p_n \in \Im$, $\cup_{i\in I} p_i \in \Im$ and for at least n elements$q_1, q_2, \dots, q_n \in \Im$, $r_1, r_2, \dots, r_n \in \Im$; $[\cup_{i\in I} q_i \notin \Im$ or $\cup_{i\in I} r_i \in_1 \Im]$.

Definition 2.3: [21]

Let (X,τ) be a NTS over X and A is subset on X. Then, the NeutroInterior of A is the union of all NeutroOpen subsets of A. Clearly, NeutroInterior of A is the biggest NeutroOpen set over X which is contained in A.

Definition 2.4:[21]

Let (X,τ) be a NTS over X and A is subset on X. Then, the NeutroClosure of A is the intersection of all NeutroClosed super sets of A. Clearly, NeutroClosure of A is the smallest NeutroClosed set over X conatining A.

3. Results

Definition 3.1.

Let X be a non-empty set endowed with two NeutroTopologies T_1andT_2. Then (X,T_1,T_2) is called a NeutroBitopological space (NBS). In this entire paper, we expressed (X,T_1,T_2) with $\Im$.

Example 3.1.

Let $X = \{1,2,3,4\}, T_1 = \{\emptyset, \{3\}, \{1,4\}, \{1,2,3\}\}$ and $T_2 = \{\emptyset, \{4\}, \{1,2\}, \{2,3\}, \{1,2,3\}\}$

For T_1: $i)$ $\emptyset \in T_1, X \notin T_1$

$ii)$ $\{3\} \cap \{1,2,3\} = \{3\} \in T_1; \{3\} \cap \{1,4\} = \emptyset \in T_1but\{1,4\} \cap \{1,2,3\} = \{1\} \notin T_1$

$iii)$ $\{3\} \cup \{1,2,3\} = \{1,2,3\} \in T_1$but $\{3\} \cup \{1,4\} = \{1,3,4\} \notin T_1$

For T_2: $i)$ $\emptyset \in T_2, X \notin T_2$

$ii)$ $\{4\} \cap \{1,2\} = \emptyset \in T_2;but\{1,2\} \cap \{2,3\} = \{2\} \notin T_2$

$iii)$ $\{1,2\} \cup \{2,3\} = \{1,2,3\} \in T_2$ but $\{4\} \cup \{1,2\} = \{1,2,4\} \notin T_2$.

Thus, it can be observed that $T_1$and T_2 are both NeutroTopologies on X. Therefore, (X,T_1,T_2) is a NeutroBitopological space. It may be noted that a NeutroBitopological (X,T_1,T_2) is not a general bitopological

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space (GBS) because T_1andT_2 are not topologies on X. Thus, a NBS is a different thing altogether and it will be seen that a NBS can be derived from any GBS. It can be seen from the following two theorems.

Proposition3.1.
If $\mathfrak{I}$ be a GBS then $(X, T_1 - \emptyset, T_2 - \emptyset)$ be a NBS.
Proof: Since the empty set is excluded from the two topologies, they are no longer general topologies but NTSs.

Proposition 3.2.
If $\mathfrak{I}$ be a GBS then $(X, T_1 - X, T_2 - X)$ be a NBS.
Proof: Since the whole set is excluded from the two topologies, they are no longer general topologies but NTSs and hence the proved.

Definition 3.2.
Let $\mathfrak{I}$ be a NBS, then the NeutroInterior of a subset A of X is defined as:

$$T_1T_2 - \text{Neu}Int(A) = T_1 - \text{Neu}Int\big(T_2 - \text{Neu}Int(A)\big).$$

Example 3.2.
Let $X = \{a, b, c, d\}$, $T_1 = \big\{\emptyset, \{a, b\}, \{c, d\}\big\}andT_2 = \big\{\emptyset, \{b\}, \{a, b\}, \{b, c\}, \{c, d\}\big\}$. Then, clearly $T_1$and T_2 are NTSs on X. So, (X, T_1, T_2) is a NBS.
Let $A = \{b, c, d\}$. Then we have, $T_1T_2 - \text{Neu}Int(A) = T_1 - \text{Neu}Int\big(T_2 - \text{Neu}Int(A)\big)$

$$= T_1 - \text{Neu}Int\big(T_2 - \text{Neu}Int(\{b, c, d\})\big)$$
$$= T_1 - \text{Neu}Int(\{b\} \cup \{b, c\} \cup \{c, d\})$$
$$= T_1 - \text{Neu}Int\{b, c, d\}$$
$$= \{c, d\}.$$

Proposition 3.3.
Let $\mathfrak{I}$ be a NBS, then $T_1T_2 - \text{Neu}Int(A) \subseteq A$.
Proof: Let $x \in T_1T_2 - \text{Neu}Int(A)$
$\Longrightarrow x \in T_1 - \text{Neu}Int(T_2 - \text{Neu}Int(A))$
$\Longrightarrow x \in T_1 - \text{Neu}Int(B)$ where, $B = T_2 - \text{Neu}Int(A) \subseteq A$
$\Longrightarrow x \in B \subseteq A$
$\Longrightarrow x \in A$
Hence, $T_1T_2 - \text{Neu}Int(A) \subseteq A$.
But the converse is not true as shown in the example below:
Example 3.3.
Let $X = \{1,2,3\}$, $T_1 = \{\emptyset, \{2\}, \{1,2\}, \{1,3\}\}$and $T_2 = \big\{\emptyset, \{1\}, \{1,3\}, \{2,3\}\big\}$. Then, (X, T_1, T_2) is aNBS. Let, $A = \{2,3\}$.
Then, we have $T_1T_2 - \text{Neu}Int(A) = T_1 - \text{Neu}Int\big(T_2 - \text{Neu}Int(\{2,3\})\big) = T_1 - \text{Neu}Int(\{2,3\}) = \{2\}$
This shows that $T_1T_2 - \text{Neu}Int(A) \subseteq A$ but $A \nsubseteq T_1T_2 - \text{Neu}Int(A)$.
So, $T_1T_2 - \text{Neu}Int(A) \neq A$.

Proposition3.4.
Let $\mathfrak{I}$ be a NBS, and $A \subseteq B$, then $T_1T_2 - \text{Neu}Int(A) \subseteq T_1T_2 - \text{Neu}Int(B)$.
Proof: Let $x \in T_1T_2 - \text{Neu}Int(A)$
$\Longrightarrow x \in T_1 - \text{Neu}Int(T_2 - \text{Neu}Int(A))$
$\Longrightarrow x \in T_1 - \text{Neu}Int(T_2 - \text{Neu}Int(B))$ since $A \subseteq B$
$\Longrightarrow x \in T_1T_2 - \text{Neu}Int(B)$
Hence, $x \in T_1T_2 - \text{Neu}Int(A) \Rightarrow x \in T_1T_2 - \text{Neu}Int(B)$.

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Proposition3.5.
Let $\Im$ be a NBS, then $T_1T_2 - \text{Neu}Int(A \cap B) \subseteq T_1T_2 - \text{Neu}Int(A) \cap T_1T_2 - \text{Neu}Int(B)$.

Proposition3.6.
Let $\Im$ be a NBS, then $T_1T_2 - \text{Neu}Int(A) \cup T_1T_2 - \text{Neu}Int(B) \subseteq T_1T_2 - \text{Neu}Int(A \cup B)$
Proof: We have $A \subseteq A \cup B \Longrightarrow T_1T_2 - \text{Neu}Int(A) \subseteq T_1T_2 - \text{Neu}Int(A \cup B)$.
Also, $B \subseteq A \cup B \Longrightarrow T_1T_2 - \text{Neu}Int(B) \subseteq T_1T_2 - \text{Neu}Int(A \cup B)$
Therefore, $T_1T_2 - \text{Neu}Int(A) \cup T_1T_2 - \text{Neu}Int(B) \subseteq T_1T_2 - \text{Neu}Int(A \cup B)$.

Remark 3.1.
Let $\Im$ be a NBS, then $T_1T_2 - \text{Neu}Int(A) \neq T_2T_1 - \text{Neu}Int(A)$.

Example 3.4.
Let $X = \{1,2,3,4\}$, $T_1 = \{\emptyset, \{1\}, \{1,2\}, \{2,3\}, \{1,2,4\}\}$ and $T_2 = \{\emptyset, \{2\}, \{1,3\}, \{2,3\}, \{2,4\}, \{1,2,3\}\}$. Let $A = \{1,2\}$.
Then, $T_1T_2 - \text{Neu}Int(A) = T_1 - \text{Neu}Int(T_2 - \text{Neu}Int(A))$
$= T_1 - \text{Neu}Int(T_2 - \text{Neu}Int(\{1,2\}) = T_1 - \text{Neu}Int(\{2\}) = \emptyset$.
And,$T_2T_1 - Int(A) = T_2 - \text{Neu}Int(T_1 - \text{Neu}Int(A)) = T_2 - \text{Neu}Int(T_1 - \text{Neu}Int(\{1,2\})$
$= T_2 - \text{Neu}Int(\{1,2\}) = \{2\}$
Therefore, $T_1T_2 - \text{Neu}Int(A) \neq T_2T_1 - \text{Neu}Int(A)$.

Remark 3.2.
Let $\Im$ be a NBS, then $T_1T_2 - \text{Neu}Int(A) = T_2T_1 - \text{Neu}Int(A)$if $T_1 = T_2$.

Remark3.3.
Let $\Im$ be a NBS, then$T_1T_2 - \text{Neu}Int\big(T_1T_2 - \text{Neu}Int(A)\big) \neq T_1T_1 - \text{Neu}Int(A)$.

Example 3.5.
Let $X = \{1,2,3,4\}$, $T_1 = \{\emptyset, \{2\}, \{1,3\}, \{2,3\}, \{2,4\}, \{1,2,3\}\}$and $T_2 = \{\emptyset, \{1\}, \{1,2\}, \{2,3\}, \{1,2,4\}\}$.
Let $A = \{1,2\}$.
Then, $T_1T_2 - \text{Neu}Int(A) = T_1 - \text{Neu}Int(T_2 - \text{Neu}Int(A))$
$= T_1 - \text{Neu}Int(T_2 - \text{Neu}Int(\{1,2\}) = T_1 - \text{Neu}Int(\{1,2\}) = \{2\}$
Now, $T_1T_2 - \text{Neu}Int\big(T_1T_2 - \text{Neu}Int(A)\big) = T_1T_2 - \text{Neu}Int(\{2\})$
$= T_1 - \text{Neu}Int(T_2 - \text{Neu}Int(\{2\}) = T_1 - \text{Neu}Int(\emptyset) = \emptyset$.

Remark 3.4.
Let $\Im$ be a NBS, then $T_1T_2 - \text{Neu}Int\big(T_1T_2 - \text{Neu}Int(A)\big) = T_1T_2 - \text{Neu}Int(A)$if $T_1 = T_2$.

Definition 3.3.
Let $\Im$ be a NBS and $A \subset X$. The intersection of all $T_1T_2 - Neutro$Closed supersets of A is called the $T_1T_2 - Neutro$Closure of A and denoted by $T_1T_2 - \text{Neu}Cl(A)$ and will be evaluated as $T_1 - \text{Neu}Cl\big(T_2 - \text{Neu}Cl(A)\big)$.

Remark 3.5.
Let $\Im$ be a NBSand $A \subset X$. Then, $T_1T_2 - \text{Neu}Cl(A) \neq T_2T_1 - \text{Neu}Cl(A)$.

Example 3.6.
Let $X = \{1,2,3,4\}$, $T_1 = \{\emptyset, \{1\}, \{1,2\}, \{1,3\}, \{1,2,4\}\}$ and $T_2 = \{\emptyset, \{2\}, \{2,3\}, \{3,4\}, \{1,2,3\}\}$.
The $T_1 - Neutro$Closed sets are: $X, \{2,3,4\}, \{3,4\}, \{2,4\}, \{3\}$
And, the $T_2 - Neutro$Closed sets are: $X, \{1,3,4\}, \{1,4\}, \{1,2\}, \{4\}\}$

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Let $A = \{3,4\}$.
Then, $T_1T_2 - \text{Neu}Cl(A) = T_1 - \text{Neu}Cl(T_2 - \text{Neu}Cl(A)) = T_1 - \text{Neu}Cl(T_2 - \text{Neu}Cl(\{3,4\})) = T_1 - \text{Neu}Cl(\{1,3,4\}) = X$
And,$T_2T_1 - \text{Neu}Cl(A) = T_2 - \text{Neu}Cl(T_1 - \text{Neu}Cl(\{3,4\})) = T_2 - \text{Neu}Cl(\{2,3,4\} \cap \{3,4\}) = T_2 - \text{Neu}Cl(\{3,4\}) = \{1,3,4\} \neq X$
Therefore, $T_1T_2 - \text{Neu}Cl(A) \neq T_2T_1 - \text{Neu}Cl(A)$.

Proposition 3.7.
Let $\mathfrak{I}$ be a NBS and $A \subset X$. If A is $T_1T_2 - \mathit{Neutro}C$losed set, then $A \subset T_1T_2 - \text{Neu}Cl(A)$.
Proof: From the definition of $T_1T_2 - \text{Neu}Cl(A)$ it is clear that $A \subset T_1T_2 - \text{Neu}Cl(A)$ since $T_1T_2 - \text{Neu}Cl(A)$ is the intersection of all supersets of A, which will obviously contain A.

Proposition 3.8.
If $A \subset B$, then $T_1T_2 - \text{Neu}Cl(A) \subset T_1T_2 - \text{Neu}Cl(B)$.
Proof: By Proposition 3.7,$B \subset T_1T_2 - \text{Neu}Cl(B)$ and $A \subset B$, so $A \subset T_1T_2 - \text{Neu}Cl(B)$ which gives $T_1T_2 - \text{Neu}Cl(A) \subset T_1T_2 - \text{Neu}Cl(B)$.

Proposition 3.9.
Let $\mathfrak{I}$ be a NBS and $A, B \subset X$. Then $T_1T_2 - \text{Neu}Cl(A \cup B) \subset T_1T_2 - \text{Neu}Cl(A) \cup T_1T_2 - \text{Neu}Cl(B)$.

Proposition 3.10.
Let $\mathfrak{I}$ be a NBS and $A, B \subset X$. Then $T_1T_2 - \text{Neu}Cl(A \cap B) \subset T_1T_2 - \text{Neu}Cl(A) \cap T_1T_2 - \text{Neu}Cl(B)$.
Proof: We have: $A \cap B \subset A \; and \; A \cap B \subset B$
Therefore, $T_1T_2 - \text{Neu}Cl(A \cap B) \subset T_1T_2 - \text{Neu}Cl(A)$ and $T_1T_2 - \text{Neu}Cl(A \cap B) \subset T_1T_2 - \text{Neu}Cl(B)$
Hence, $T_1T_2 - \text{Neu}Cl(A \cap B) \subset T_1T_2 - \text{Neu}Cl(A) \cap T_1T_2 - \text{Neu}Cl(B)$.

Proposition 3.11.
Let $\mathfrak{I}$ be a NBS and $A \subset X$. Then $T_1T_2 - \text{Neu}Cl(T_1T_2 - \text{Neu}Cl(A)) = T_1T_2 - \text{Neu}Cl(A)$ if A is $T_1T_2 - \mathit{Neutro}C$losed.
Proof: If A is$T_1T_2 - \mathit{Neutro}C$losed, then A is the smallest $\mathit{Neutro}C$losed set containing A, so $T_1T_2 - \text{NeuCl(A)} = \text{A}$.
Therefore, $T_1T_2 - \text{Neu}Cl(T_1T_2 - \text{Neu}Cl(A)) = T_1T_2 - \text{Neu}Cl(A)$.

Proposition 3.12.
Let $\mathfrak{I}$ be a NBS and $A \subset X$, then the NeutoInterior of A is equal to the complement of the NeutroClosureof the complement of A.

Proposition 3.13.
Let $\mathfrak{I}$ be a NBSand $A \subset X$, then the NeutroClosure of the complement of A is not equal to the complement of the NeutroInterior of A.

Proposition 3.14.
Let $\mathfrak{I}$ be a NBS and $A \subset X$, then the NeutroClosure of A is equal to the complement of the NeutroInterior of the complement of A.

Definition 3.4.
Let $\mathfrak{I}$ be a NBS and $A \subset X$. A point and $x \in X$ is said to be $T_1T_2 - \mathit{NeutotroExterior}$ of Aif $x \in T_1T_2 - \text{Neu}Int(A^c)$.

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Definition 3.5.

Let $\Im$ be a NBS and $A \subset X$. A point and $x \in X$ is said to be a $T_1T_2 - NeutroBoundary$ point if it is neither an $T_1T_2 - NeutroInterior$ nor $T_1T_2 - NeutroExterior$ point of A.

We define $T_1T_2 - \text{Neu}Bd(A) = T_1T_2 - \text{Neu}Cl(A) \cap T_1T_2 - \text{Neu}Cl(A^c)$.

Proposition 3.15.

Let $\Im$ be a NBS with $T_1 = T_2$and $A, B \subset X$. Then the following results are found:

(i) $T_1T_2 - \text{Neu}Bd\big(T_1T_2 - \text{Neu}Int(A)\big) \subseteq T_1T_2 - \text{Neu}Bd(A)$

(ii) $T_1T_2 - \text{Neu}Bd\big(T_1T_2 - \text{Neu}Cl(A)\big) \subseteq T_1T_2 - \text{Neu}Bd(A)$

(iii) $T_1T_2 - \text{Neu}Bd(A \cup B) \subseteq T_1T_2 - \text{Neu}Bd(A) \cup T_1T_2 - \text{Neu}Bd(B)$

(iv) $T_1T_2 - \text{Neu}Bd(A \cap B) \subseteq T_1T_2 - \text{Neu}Bd(A) \cup T_1T_2 - \text{Neu}Bd(B)$.

Remark 3.6.

If $T_1 \neq T_2$,then the **proposition 3.15** is not true that is for $A, B \subset X$, the following results are found:

(i) $T_1T_2 - \text{Neu}Bd\big(T_1T_2 - \text{Neu}Int(A)\big) \nsubseteq T_1T_2 - \text{Neu}Bd(A)$

(ii) $T_1T_2 - \text{Neu}Bd\big(T_1T_2 - \text{Neu}Cl(A)\big) \nsubseteq T_1T_2 - \text{Neu}Bd(A)$

(iii) $T_1T_2 - \text{Neu}Bd(A \cup B) \nsubseteq T_1T_2 - \text{Neu}Bd(A) \cup T_1T_2 - \text{Neu}Bd(B)$

(iv) $T_1T_2 - \text{Neu}Bd(A \cap B) \nsubseteq T_1T_2 - \text{Neu}Bd(A) \cup T_1T_2 - \text{Neu}Bd(B)$

For this we cite the following examples

(i) **Example 3.7.**

Let $X = \{a, b, c, d\}$, $T_1 = \big\{\emptyset, \{a\}, \{c\}, \{a, b\}, \{b, c\}, \{a, b, d\}\big\}$and $T_2 = \{\emptyset, \{b\}, \{d\}, \{c, d\}, \{a, d\}, \{a, b, c\}\}$

$T_1 - NeutroClosed$ sets are: $X, \{b, c, d\}, \{a, b, d\}, \{c, d\}, \{a, d\}, \{c\}$ and

$T_2 - NeutroClosed$ sets are are: $X, \{a, c, d\}, \{a, b, c\}, \{a, b\}, \{b, c\}, \{d\}$

Let $A = \{a, d\}$, $A^C = \{b, c\}$.

Now $T_1T_2 - \text{Neu}Int(A) = \{a\} = B(say)$.

Now $T_1T_2 - \text{Neu}Cl(B) = \{a, d\}$and $T_1T_2 - \text{Neu}Cl(B^C) = X$. Therefore, $T_1T_2 - \text{Neu}Bd(B) = \{a, d\}$.

Again $T_1T_2 - \text{Neu}Cl(A) = X$ and $T_1T_2 - \text{Neu}Cl(A^C) = \{b, c, d\}$, and $T_1T_2 - \text{Neu}Bd(B) = \{b, c, d\}$

Hence $T_1T_2 - \text{Neu}Bd\big(T_1T_2 - \text{Neu}Int(A)\big) \nsubseteq T_1T_2 - \text{Neu}Bd(A)$.

Similarly, other properties of Remark 3.6 can be shown.

5. Conclusions

In this work, NeutroBitological space is studied. The terms NeutroInterior, NeutroClosureand NeutroBoundary are defined. It is seen that some properties of NBS are not the same as the properties of GBS. For this, we have cited examples. We hope, this work can lead towards the development of many properties of NeutroBitological space.

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Chapter Two

A Study on the Properties of AntiTopological Space

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ABSTRACT

In the current study, the properties of Interior, Closure and Boundary points of the antiTopological studies have been observed and studied by introducing the ideas of AntiInterior, AntiClosure, and AntiBoundary. It has been found that some of the properties that are valid in general topological spaces are also valid in anti-topological spaces while some of the properties are found to be not valid, as in the case that the AntiInterior of a set is not the smallest closed set that contains the set as in the general topological spaces.

Keywords: AntiInterior, AntiClosure, AntiBoundary, AntiTopological space.

INTRODUCTION

General topology is the branch where most of the studies had been done by all the founders of topology and the various properties that the subsets of the topology have, like continuity, connectedness, compactness, etc. But, most of the properties that have been accepted to be of the topological spaces are, as put forward by the ones who defined them, without any actual testing on whether they apply to the real-world situations, whether they are true for all cases or whether there may exist some cases where those cases are not applicable in general. That is, where the proposal of a fuzzy set came in 1965 by Lofti A. Zadeh [39], and it is where elements of a set are assigned degree of membership and degree of non-membership. And, in due course of time, the case of neutrosophy had to be ushered in by Florentine Smarandache in 1998. The neutrosophic set encompasses three components, namely the truth (T), the indeterminacy (I), and the falsity (F) of a statement or a property. Many authors (Sahin *et al.* [40, 41, 56, 57], Hassan *et al.* [42], Ulucay *et al.* [43-45, 48-50], Broumi *et al.* [46]) applied the concepts of the neutrosophic set to various field [58-84]. The present study deals with the falsity component of the neutrosophic set. Anti-topological space was defined along with neutro-topological space by Sahin *et al.* [25].

In recent years, there has been a surge in academic interest in neutrosophic set theory. The concept of neutro-structures and anti-structures was first defined by Florentin Smarandache [30, 31]. Also, a lot of researchers studied neutroalgebra [51-55]. Şahin *et al.* [25] discussed the idea of neutro-topological space and anti-topological space. Smarandache [33] studied NeutroAlgebra as a generalization of partial algebra. Agboola [1-3] investigated the idea of NeutroRings, NeutroGroups, and finite NeutroGroups of type-NG.

Smarandache [34] proposed the generalizations and alternatives of Classical Algebraic Structures to NeutroAlgebraic Structures and AntiAlgebraic Structures. Al-Tahan *et al.* [6] studied the NeutroOrderedAlgebra, NeutroHyper structures, and their properties.

Smarandache [30-31] founded and studied the concept of neutro-structures and anti-structures. From the concepts of NeutroAlgebra, he showed that if a statement (theorem, lemma, consequence, property, etc.) is totally true in a classical Algebra, it does not mean that it is also totally true in a NeutroAlgebra or in an AntiAlgebra. It depends on the operations and axioms (if they are totally true, partially true, totally false, or partially or totally indeterminate) it is based upon.

For examples:

(1) Let $(A,*)$ be a NeutroAlgebra (it has NeutroOperations or NeutroAxioms while the others are classical Operations and classical Axioms, and no AntiOperation and no AntiAxiom).

Statement: If x, y in A, then $x * y$ in A.
This statement is true for classical Algebra.
But for a NeutroAlgebra we have:

(a) The Statement is true if the operation $*$ is a classical Operation (totally true).
(b) The Statement is true if the operation $*$ is a NeutroAxiom,
but x, y both belong to the partially true subset;
(c) The Statement is false if the operation $*$ is a NeutroAxiom,
and at least one of x or y belongs to the partially false subset.

(2) Similarly, for the NeutroGroup.
Let A be a NeutroGroup, and x in A. Then its inverse x^{-1} is also in A. This is true for the classical Group.
For the NeutroGroup:
(a) This is true if the inverse element axiom is totally true;
(b) This is true if the NeutroInverse element axiom is partially true, and x belongs to the true subset;
(c) This is false otherwise.

By observing the above concepts, the properties of Interior, Closure and Boundary points of the AntiTopological studies have been observed.

BACKGROUND

Definition 2.1: [34] The NeutroSophication of the Law

(i) Let X be a non-empty set and $*$ be binary operation. For some elements $(a, b) \in (X, X)$, $(a * b) \in X$ (degree of well defined (T)) and for other elements $(x, y), (p, q) \in (X, X)$; [$x * y$ is indeterminate (degree of indeterminacy (I)), or $p * q \notin X$ (degree of outer-defined (F)],where (T, I, F) is different from (1,0,0) that represents the Classical Law, and from (0,0,1) that represents the AntiLaw.

(ii) In NeutroAlgebra, the classical well-defined for $*$ binary operation is divided into three regions: degree of well-defined (T), degree of indeterminacy (I) and degree of outer-defined (F) similar to neutrosophic set and neutrosophic logic.

Definition 2.2: [25] Let X be the non-empty set and τ be a collection of subsets of X. Then τ is said to be a NeutroTopology on X and the pair (X, τ) is said to be a NeutroTopological space, if at least one of the following conditions hold good:

(i) $[(\emptyset_N \in \tau, X_N \notin \tau)\ or\ (X_N \in \tau, \emptyset_N \notin \tau)\ ]$ or $[\emptyset_N, X_N \in_{\sim} \tau]$.

(ii) For some n elements $a_1, a_2, \dots, a_n \in \tau, \bigcap_{i=1}^{n} a_i \in \tau$ [degree of truth T] and for other n elements $b_1, b_2, \dots, b_n \in \tau, p_1, p_2, \dots, p_n \in \tau$; $[(\bigcap_{i=1}^{n} b_i \notin \tau)$ [degree of falsehood F] or $(\bigcap_{i=1}^{n} p_i$ is indeterminate (degree of indeterminacy I)], where n is finite; where (T, I, F) is different from (1,0,0) that represents the Classical Axiom, and from (0,0,1) that represents the AntiAxiom.)].

(iii) For some n elements $a_1, a_2, \dots, a_n \in \tau, \bigcup_{i=1} a_i \in \tau$ [degree of truth T] and for other n elements $b_1, b_2, \dots, b_n \in \tau, p_1, p_2, \dots, p_n \in \tau$; $[(\bigcup_{i=I} b_i \notin \tau)$ [degree of falsehood F] or $(\bigcup_{i=I} p_i$ is indeterminate (degree of indeterminacy I)], where n is finite; where (T, I, F) is different from (1,0,0) that represents the Classical Axiom, and from (0,0,1) that represents the AntiAxiom.)].

Remark 2.1: [25] The symbol "$\in_{\sim}$" will be used for situations where it is an unclear appurtenance (not sure if an element belongs or not to a set). For example, if it is not certain whether "a" is a member of the set P, then it is denoted by a $\in_{\sim} P$.

Theorem 2.1: [25] Let (X, τ) be a classical topological space. Then $(X, \tau - \emptyset)$ is a NeutroTopological space.

Theorem 2.2: [25] Let (X, τ) be a classical topological space. Then $(X, \tau - X)$ is a NeutroTopological space.

Definition 2.3: [34]**: The Anti-sophication of the Law** (totally outer-defined)

Let X be a non-empty set and * be a binary operation. For all double elements $(x, y) \in (X, X), x * y \notin X$ (totally outer-defined).

Definition 2.4: [25]: AntiTopological space: Let X be a non-empty set, τbe a collection of subsets of X. If the following conditions {i, ii, iii} are satisfied then, $\tau$is called an anti-topology and (X, τ) is called an anti-topological space.

i) $\emptyset, X \notin \tau$

ii) For all $q_1, q_2, q_3, \dots, q_n \in \tau,\ \bigcap_{i=1}^{n} q_i \notin \tau$, where n is finite.

iii) For all $q_1, q_2, q_3, \dots, q_n \in \tau,\ \bigcup_{i \in I} q_i \notin \tau$, where I is an index set.

MAIN FOCUS OF THE CHAPTER

Proposition 3.1: In an AntiTopological space. The following conditions (i), (ii), and (iii) are satisfied.

(i) Empty set and X is not AntiOpen.

(ii) Union of the AntiOpen sets is not AntiOpen.

(iii) Intersection of the AntiOpen sets is not AntiOpen.

Examples 3.1: Let $X = \{a, b, c, d\}$ and $\tau = \{\{a, b\}, \{c, d\}, \{b, c\}\}$. Then (X, τ) is antiTopological space.

(i) Here $\emptyset\ and\ X$ are not AntiOpen.

(ii) $\{a, b\} \cup \{c, d\} = \{a, b, c, d\}; \{a, b\} \cup \{b, c\} = \{a, b, c\}; \{c, d\} \cup \{b, c\} = \{b, c, d\}$ which are all not AntiOpen in (X, τ).

(iii) Also,$\{a, b\} \cap \{c, d\} = \emptyset; \{a, b\} \cap \{b, c\} = \{b\}; \{c, d\} \cap \{b, c\} = \{c\}$, which are all not AntiOpen in (X, τ).

Definition 3.1: Let (X, τ) be anAntiTopological space over X and A is subset on X. Then, the AntiInterior of A is the union of all AntiOpen subsets of A. Clearly, AntiInterior of A is the biggest AntiOpen set over X which is contained A.

That is, $AntiInt(A) = \cup \{B, where\ B\ is\ open\ and\ B \subseteq A\}$

Proposition 3.2: Let (X, τ) be an AntiTopological space over X and A is subset on X. If A is AntiOpen, then $AntiInt(A) = A$.

Proof: By definition, $AntiInt(A) = \cup \{B, where\ B\ is\ open\ and\ B \subseteq A\}$.

If A is AntiOpen, and $B \subset A$ and B is AntiOpen then $A \cap B = B$ and it will violate the condition (iii) of the definition of the AntiTopological Spaces. Hence, $B \not\subset A$. So $B = A$. Hence, $AntiInt(A) = A$.

Proposition 3.3: In an AntiTopological space (X, τ), $AntiInt(A) \notin \tau$ if A is not AntiOpen.

Proof: By definition, $AntiInt(A) = \cup \{B, where\ B\ is\ AntiOpen\ and\ B \subseteq A\}$.

By **Proposition 3.2**, if A is AntiOpen, then $AntiInt(A) = A$. If A is not AntiOpen, then either $AntiInt(A) = \emptyset$ or, $AntiInt(A) = B \cup C$, where B and C are AntiOpen. And $B \cup C$ cannot be contained in τ otherwise it will violate condition (ii) of the Proposition 3.1.

Example 3.2: Let $X = \{1,2,3,4\}$ and $\tau = \{\{1,2\}, \{2,3\}, \{3,4\}\}$.
Let $A = \{1,2,3\}$, then $AntiInt(A) = \{1,2\} \cup \{2,3\} = \{1,2,3\} \notin \tau$.
And, $A = \{2,4\}$, then $AntiInt(A) = \emptyset \notin \tau$.
Observation: From Example 3.2, it is observed that *AntiInt(A)* is equal to A even if A is not AntiOpen.

Proposition 3.4: Let (X, τ) be AntiTopological space. Then

(i) $A \subseteq B \Rightarrow AntiInt(A) \subseteq AntiInt(B)$
(ii) $AntiInt(A \cap B) \subseteq AntiInt(A) \cap AntiInt(B)$
(iii) $AntiInt(A) \cup AntiInt(B) \subseteq AntiInt(A \cup B)$
(iv) $AntiInt(AntiInt(A)) = AntiInt(A)$ if A is AntiOpen.

Proof:

(i) Both A and B cannot be AntiOpen at the same time because in that case A cannot be a subset of B. Suppose that A is AntiOpen, and B is not. Then, $AntiInt(A) = A$ and $AntiInt(B) = \{A \cup C_i\}$ since $A \subseteq B$ and A is AntiOpen, where C_i are AntiOpen. Hence, $AntiInt(A) \subseteq AntiInt(B)$ in this case. Next, suppose that B is AntiOpen while A is not, then $AntiInt(B) = B$ and $AntiInt(A) = \cup \{C, C \text{ is AntiOpen}\}$ and $C \neq B$. By Proposition 3.3, $AntiInt(A) \notin \tau$ and $A \subseteq B$. The only possibility for this is that $AntiInt(A) = \emptyset$.

(ii) For any A and B, $A \cap B \subseteq A$ and $A \cap B \subseteq B$.

So, we have: $AntiInt(A \cap B) \subseteq AntiInt(A)$ and $AntiInt(A \cap B) \subseteq AntiInt(B)$

Hence, $AntiInt(A \cap B) \subseteq AntiInt(A) \cap AntiInt(B)$

(iii) For any A and B, $A \subseteq A \cup B$ and $B \subseteq A \cup B$.

So, we have: $AntiInt(A) \subseteq AntiInt(A \cup B)$ and $AntiInt(B) \subseteq AntiInt(A \cup B)$

Hence, $AntiInt(A) \cup AntiInt(B) \subseteq AntiInt(A \cup B)$

(iv) The proof is direct by Proposition 3.2.

Definition 3.2: Let (X, τ) be an AntiTopological space and a subset A of X is said to be τ-AntiClosed set if and only if its complement A^C is an AntiOpen set.

Proposition 3.5: In an AntiTopological space. The conditions (i) and (ii) are satisfied.

(i) The intersection of AntiClosed sets isnot AntiClosed.

(ii) Union of AntiClosed sets is notAntiClosed.

Definition 3.3: Let (X, τ) be anAntiTopological space over X and A is subset on X. Then, the AntiClosure of A is the intersection of all AntiClosed super sets of A. Clearly, AntiClosure of A is *not* the smallest AntiClosed set over X containing A, which is shown in the Proposition 3.6 (ii) below.

That is, $AntiCl(A) = \cap \{G: G \supseteq A$ and G is AntiClosed$\}$

Example 3.3: Let $X = \{1,2,3,4,5\}$ and $\tau = \{\{1\}, \{2\}, \{3\}, \{5\}\}$. Then, the AntiClosed sets are: $\{2,3,4,5\}, \{1,3,4,5\}, \{1,2,4,5\}$ and $\{1,2,3,4\}$. Let $A = \{1,2\}$, then $AntiCl(A) = \{1,2,4,5\} \cap \{1,2,3,4\} = \{1,2,4\}$.

Proposition 3.6: Let (X, τ) be an AntiTopological space. Then

(i) $AntiCl(A)$ is not the smallest AntiClosed set containing A.

(ii) If A is AntiClosed, then $A = AntiCl(A)$.

Proof:

(i) We prove it by a counter example. Let $X = \{1,2,3,4,5\}$ and $\tau = \{\{1\}, \{2\}, \{3\}, \{5\}\}$. Then, the AntiClosed sets are: $\{2,3,4,5\}, \{1,3,4,5\}, \{1,2,4,5\}$ and $\{1,2,3,4\}$. Let $A = \{1,2\}$, then $AntiCl(A) = \{1,2,4,5\} \cap \{1,2,3,4\} = \{1,2,4\}$ which is not AntiClosed.

We may consider another example by considering A as an AntiOpen set, say $A = \{1\}$, then $AntiCl(A) = \{1,3,4,5\} \cap \{1,2,4,5\} \cap \{1,2,3,4\} = \{1,4\}$ which is also not AntiClosed.

Thus, AntiClosure of A is *not* the smallest AntiClosed set over X containing A.

(ii) Proof is obvious from the definition of anti-topology.

Proposition 3.7: Let (X, τ) be AntiTopological space and let $A, B \subseteq X$. If B is AntiClosed, then

(i) $A \subseteq AntiCl(A)$

(ii) $A \subseteq B \implies AntiCl(A) \subseteq AntiCl(B)$

(iii) $AntiCl(A) \cup AntiCl(B) \subseteq AntiCl(A \cup B)$

(iv) $AntiCl(A \cap B) \subseteq AntiCl(A) \cap AntiCl(B)$

(v) $AntiCl\big(AntiCl(B)\big) = AntiCl(B)$

Proof:

(i) By definition, we have $AntiCl(A)$ is a set containing A. So, $A \subseteq AntiCl(A)$

(ii) If B is closed then, $AntiCl(B) = B$. Thus, $A \subseteq B \Rightarrow$ A $\subseteq AntiCl(B)$ which give, $AntiCl(A) \subseteq AntiCl(B)$.

(iii) $A \subseteq A \cup B \Rightarrow AntiCl(A) \subseteq AntiCl(A \cup B)$ by (i) above. Also, $B \subseteq A \cup B \Rightarrow AntiCl(B) \subseteq AntiCl(A \cup B)$ by (i) above. Hence, $AntiCl(A) \cup AntiCl(B) \subseteq AntiCl(A \cup B)$

(iv) $A \cap B \subseteq A \Rightarrow$ AntiCl(A ∩ B) $\subseteq AntiCl(A)$ by (i) above. Also, $A \cap B \subseteq B \Rightarrow$ AntiCl(A ∩ B) $\subseteq AntiCl(B)$ by (i) above. Hence, $AntiCl(A \cap B) \subseteq AntiCl(A) \cap AntiCl(B)$.

(v) Since B is AntiClosed, we have $AntiCl(B) = B$. So, $AntiCl\big(AntiCl(B)\big) = AntiCl(B)$

Remark 3.1: In Proposition 3.7, if B is not AntiClosed then the results are not generally true. It is because the AntiClosure of every subset of X will not always exist because X is not AntiClosed.

Proposition 3.8: Let (X,τ) be AntiTopological space and let $A \subseteq X$. Then

(i) $AntiInt(A) = \left(AntiCl(A^C)\right)^C$, if A is AntiOpen.

(ii) $AntiCl(A^C) = \left(AntiInt(A)\right)^C$, if A is AntiOpen.

(iii) $AntiCl(A) = \left(AntiInt(A^C)\right)^C$, if A is AntiClosed.

(iv) $AntiCl\big(AntiInt(A)\big) = AntiCl(A)$

Proof:

(i) Let $x \in AntiInt(A) \Rightarrow x \in A \Rightarrow x \notin A^C \Rightarrow x \notin AntiCl(A^C) \Rightarrow x \in \left(AntiCl(A^C)\right)^C$.

Hence, $AntiInt(A) \subseteq \left(AntiCl(A^C)\right)^C$.

Conversely, let $x \in \left(AntiCl(A^C)\right)^C \Rightarrow x \notin AntiCl(A^C) \Rightarrow x \notin A^C \Rightarrow x \in A$.

Hence, $x \in AntiInt(A)$. So, $\left(AntiCl(A^C)\right)^C \subseteq AntiInt(A)$.

(ii) Let $x \in AntiCl(A^C) \Leftrightarrow x \in A^C \Leftrightarrow x \notin A \Leftrightarrow x \notin AntiInt(A) \Leftrightarrow x \in \left(AntiInt(A)\right)^C$.

Hence, $AntiCl(A^C) = \left(AntiInt(A)\right)^C$.

(iii) Let $x \in AntiCl(A) \Leftrightarrow x \in A \Leftrightarrow x \notin A^C \Leftrightarrow x \notin AntiInt(A^C) \Leftrightarrow x \in \left(AntiInt(A^C)\right)^C$.

Hence, $AntiCl(A) = \left(AntiInt(A^C)\right)^C$.

(iv) Let $x \in AntiCl\big(AntiInt(A)\big) \Leftrightarrow x \in AntiInt(A) \Leftrightarrow x \in A \Leftrightarrow x \in AntiCl(A)$.

Hence, $AntiCl\big(AntiInt(A)\big) = AntiCl(A)$.

Definition 3.4: Let (X,τ) be an AntiTopological space over X and A is subset on X. Then AntiBoundary of A is defined as $AntiBd(A) = AntiCl(A) \cap AntiCl(A^C)$.

Example 3.4: Let $X = \{1,2,3,4\}$ and $\tau = \big\{\{2\},\{3\}\big\}$. Then, the AntiClosed sets are: $\{1,3,4\}$ and $\{1,2,4\}$.

Let $A = \{3\}$, then $A^c = \{1,2,4\}$. Now, $AntiCl(A) = \{1,3,4\}$ and $AntiCl(A^C) = \{1,2,4\}$. So, the $AntiBd(A) = AntiCl(A) \cap AntiCl(A^C) = \{1,3,4\} \cap \{1,2,4\} = \{1,4\}$.

Proposition 3.9: Let (X,τ) be Anti-Topological space and let $A, B \subseteq X$. Then

(i) $AntiCl(A) - AntiInt(A) = AntiBd(A)$

(ii) $AntiInt(A) = A - AntiBd(A)$

(iii) $AntiInt(A) \cup AntiInt(A^C) = [AntiBd(A)]^C$

(iv) $AntiBd\big(Int(A)\big) = AntiBd(A)$

(v) $AntiBd\big(AntiCl(A)\big) \subseteq AntiBd(A)$

(vi) $AntiBd(A \cup B) \subseteq AntiBd(A) \cup AntiBd(B)$

(vii) $Bd(A \cap B) \subseteq Bd(A) \cup Bd(B)$.

Proof:

(i) Let $x \in AntiCl(A) - AntiInt(A)$
Now,

$x \in AntiCl(A) - AntiInt(A)$
$\Leftrightarrow x \in AntiCl(A)\ and\ x \notin AntiInt(A)$
$\Leftrightarrow x \in AntiCl(A)\ and\ x \notin A$
$\Leftrightarrow x \in AntiCl(A)\ and\ x \in A^C$
$\Leftrightarrow x \in AntiCl(A)\ and\ x \in AntiCl(A^C)$
$\Leftrightarrow x \in AntiCl(A) \cap AntiCl(A^C)$
$\Leftrightarrow x \in AntiBd(A)$.
Hence, $AntiCl(A) - AntiInt(A) = AntiBd(A)$.

(ii) Let $x \in AntiInt(A)$
Now,
$x \in AntiInt(A)$
$\Leftrightarrow x \in A\ and\ x \notin A^C$
$\Leftrightarrow x \in A\ and\ x \in AntiCl(A)\ and\ \ x \notin AntiCl(A^C)$
$\Leftrightarrow x \in A\ and\ x \notin AntiBd(A)$
$\Leftrightarrow x \in A - Bd(A)$
Hence, $AntiInt(A) = A - AntiBd(A)$.

(iii) From definition, we have
$AntiBd(A) = AntiCl(A) \cap AntiCl(A^C)$
$\Leftrightarrow [AntiBd(A)]^C = [AntiCl(A) \cap AntiCl(A^C)]^C$
$\Leftrightarrow [AntiBd(A)]^C = [AntiCl(A)]^C \cup [AntiCl(A^C)]^C$
$\Leftrightarrow [AntiBd(A)]^C = AntiInt(A^C) \cup AntiInt(A)$, by Proposition 3.8.
Hence, $AntiInt(A) \cup AntiInt(A^C) = [AntiBd(A)]^C$

(iv) $AntiBd\big(AntiInt(A)\big) = AntiCl\big(AntiInt(A)\big) \cap AntiCl\left[\big(AntiInt(A)\big)^C\right]$ [by Proposition 3.8 (i)]
$= AntiCl\big(AntiInt(A)\big) \cap AntiCl\left[\left\{\big(AntiCl(A^C)\big)^C\right\}^C\right]$ $\left[as\ \big(AntiCl(A^C)\big)^C = AntiInt(A)\right]$
$= AntiCl\big(AntiInt(A)\big) \cap AntiCl(A^C)[as\ (P^C)^C$
$= P\ and\ AntiCl\big(AntiCl(P)\big) = AntiCl(P)$, for any set P
$= AntiCl(A) \cap AntiCl(A^C)$ [by Proposition 3.8 (iv)]
$= AntiBd(A)$ [by definition]
Hence, $AntiBd\big(AntiInt(A)\big) = AntiBd(A)$.

(v) $AntiBd(AntiCl(A)) = AntiCl\big(AntiCl(A)\big) \cap AntiCl\left[\big(AntiCl(A)\big)^C\right]$
Now, $A \subseteq AntiCl(A) \Rightarrow \big(AntiCl(A)\big)^C \subseteq A^C$
$\Rightarrow AntiCl\left[\big(AntiCl(A)\big)^C\right] \subseteq AntiCl(A^C)$ $[A \subseteq B \Rightarrow AntiCl(A) \subseteq AntiCl(B)]$
Hence, $AntiBd\big(AntiCl(A)\big) \subseteq AntiCl(A) \cap AntiCl(A^C) = AntiBd(A)$
i.e., $AntiBd\big(AntiCl(A)\big) \subseteq AntiBd(A)$.

(vi) $AntiBd(A \cup B) = AntiCl(A \cup B) \cap AntiCl(A \cup B)^C$
$\subseteq [AntiCl(A) \cup AntiCl(B)] \cap [AntiCl(A^C) \cap AntiCl(B^C)]$
$= [AntiCl(A) \cap \{AntiCl(A^C) \cap AntiCl(B^C)\}] \cup [AntiCl(B) \cap \{AntiCl(A^C) \cap AntiCl(B^C)\}]$
$= [\{AntiCl(A) \cap AntiCl(A^C)\} \cap AntiCl(B^C)] \cup [\{AntiCl(B) \cap AntiCl(B^C)\} \cap AntiCl(A^C)]$
$= [AntiBd(A) \cap AntiCl(B^C)] \cup [AntiBd(B) \cap AntiCl(A^C)]$
$\subseteq AntiBd(A) \cup AntiBd(B)$
Hence, $AntiBd(A \cup B) \subseteq AntiBd(A) \cup AntiBd(B)$.

(vii) $AntiBd(A \cap B) = AntiCl(A \cap B) \cap AntiCl[(A \cap B)^C]$

$\subseteq [AntiCl(A) \cap AntiCl(B)] \cap AntiCl(A^C \cup B^C)$

$= [AntiCl(A) \cap AntiCl(B)] \cap [AntiCl(A^C) \cup AntiCl(B^C)]$

$= [\{AntiCl(A) \cap AntiCl(B)\} \cap AntiCl(A^C)] \cup [\{AntiCl(A) \cap AntiCl(B)\} \cap AntiCl(B^C)]$

$= [\{AntiCl(A) \cap AntiCl(A^C)\} \cap AntiCl(B)] \cup [AntiCl(A) \cap \{AntiCl(B) \cap AntiCl(B^C)\}]$

$= [AntiBd(A) \cap AntiCl(B)] \cup [AntiCl(A) \cap AntiBd(B)]$

$\subseteq AntiBd(A) \cup AntiBd(B)$

Hence, $AntiBd(A \cap B) \subseteq AntiBd(A) \cup AntiBd(B)$.

Proposition 3.10: Let (X, τ) be AntiTopological space and let $A \subseteq X$. If A is AntiOpen, then $AntiCl(A) - A = AntiBd(A)$

Proof: Since A is AntiOpen, therefore $AntiInt(A) = A$ [from Proposition 3.2]

and $AntiInt(A) = \left(AntiCl(A^C)\right)^C$ [from Proposition 3.8 (i)]

$AntiCl(A) - A = AntiCl(A) - Int(A)$

$= AntiCl(A) - \left(AntiCl(A^C)\right)^C$

$= AntiCl(A) \cap \left\{\left(AntiCl(A^C)\right)^C\right\}^C$

$= AntiCl(A) \cap AntiCl(A^C)$

$= AntiBd(A)$.

Hence $AntiCl(A) - A = AntiBd(A)$.

Remark 3.2: If the subset A of X is not AntiOpen, then the equality in Proposition 3.10 may not hold. We will show it by an example:

Let $X = \{1,2,3,4,5\}$ and $\tau = \{\{3\}, \{1,2\}, \{1,4\}, \{4,5\}\}$.

The AntiClosed sets are: $\{1,2,4,5\}, \{3,4,5\}, \{2,3,5\}, \{1,2,3\}$.

Let A= $\{1,3\}$, then $A^c = \{2,4,5\}$.

Now, $AntiCl(A) = \{1,2,3\}$ and $AntiCl(A^c) = \{1,2,4,5\}$.

So, $AntiBd(A) = AntiCl(A) \cap AntiCl(A^c) = \{1,2\}$.

Now, $AntiCl(A) - A = \{1,2,3\} - \{1,3\} = \{2\} \neq AntiBd(A)$.

Conclusion

In this study, it is observed that many properties of AntiTopological space are not the same as general topological space and NeutroTopological Spaces. Then we have investigated the properties of the interior, closure, and boundary of AntiTopological spaces. Hope our work will help in further study of AntiTopological space. This may lead to a new beginning for further research on the study of Topological space.

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A Study on Continuity functions in Neutro-Topological Spaces

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Abstract: The specific purpose of this study is to define continuity of mappings in neutro-topological spaces using neutro-open and neutro-closed sets and analyze the properties of continuous functions that are true in classical topological spaces in the neutro-topological space. Neutro-interior and neutro-closure in neutro-topological spaces have some properties that are somewhat different from those in classical topological spaces. However, with the definition of a new form of continuity, termed as weakly neutro-continuity, much of the properties of continuous functions could be established in neutro-topological spaces. Neutro-open map and neutro-closed maps are also defined on the basis of neutro-open and neutro-closed sets. The notion of weakly neutro-continuity has been used to define neutro-homeomorphism and many of the properties of homeomorphism are analyzed and found to be true in the case of neutro-homeomorphism. A comparison of some of the properties of continuity and homeomorphism in classical topological spaces have been done vis-à-vis the neutrosophic topological spaces and neutro-topological spaces.

Keywords: Neutro-topological space, neutro-continuity, weakly neutro-continuity, neutro-open map, neutro-closed map, neutro-homeomorphism

1. Introduction

Continuity of functions in topological spaces is a well-established notion. Kelley [1] characterized continuity of functions by eight equivalent definitions and characterizations. The validity of any one of the characterizations is equivalent for a function to be continuous. Halfar [2] in 1960 studied conditions that imply continuity of functions between spaces and studied continuity of functions in terms of connectedness and compactness of the spaces. Levine [3-4] introduced the concept of weakly continuity. In the later article he introduced semi-open sets and he defined semi-continuity of functions in terms of the semi-open sets. Other studies on semi-continuity can be seen in [5-7]. Studies on weakly, sub-weakly and semi-weakly continuous functions can be seen in [9-12]. Hussain [13] defined almost continuity of functions by the openness of image of inverse image of the neighborhood of a point. Almost continuity has been studied by many other scholars [14-18]. Gentry *et al*. [19] introduced somewhat open sets and defined somewhat continuous functions and studied the properties of such continuous functions. Noiri [20] introduced δ-closed sets and its complement, the δ-open sets and subsequently defined the δ-continuous functions and proved that δ-continuity implies weakly continuity. Mashour *et al*. [21] defined pre-continuous and weakly pre-continuous functions on the basis of pre-open sets. Further, Mashour *et al*. [22] defined α-open sets, and introduced α-continuous functions and α-open maps. More studies on the α-continuous functions have been done in [23-25]. Many other scholars defined many different types of open sets and defined continuity with regard to those open sets and the list is huge as new kinds of open sets and new kinds of closed sets have been defined over the years. Zadeh [26] defined the fuzzy set with the aim to overcome the shortfall of the Cantorian set in failing to provide a base to study ambiguity. Chang [27] defined fuzzy topological space based on the fuzzy set defined by Zadeh. Atanassov [28] refined the fuzzy set by introducing intuitionistic fuzzy set. Coker [29] defined intuitionistic fuzzy topological space. Smarandache [30] introduced the neutrosophic logic. Further, Smarandache [31] refined the fuzzy set and hence the intuitionistic fuzzy set by bringing about the neutrosophic set. The neutrosophic set has been defined to encompass three parts: "the Truth, the Indeterminacy and the Falsity" of an entity to belong to a set. The fuzzy set is based only on the grade of truth while the intuitionistic fuzzy set is based on both the grade of truth as well as the grade of falsity. Thus the third component indeterminacy refined the study of the fuzzy set. Further, Salama *et al.* [32] defined the neutrosophic topological space. Salama *et al.* [33] defined neutrosophic closed set and defined different types of neutrosophic continuity based on the neutrosophic closed sets and proposed many results on neutrosophic continuity. Al-Omeri *et al*. [34] defined different types of neutrosophic open sets in neutrosophic topological spaces and studied neutrosophic continuity on the basis of the new sets that they defined. Senyurt *et al*. [35] studied neutrosophic continuity by establishing the properties of continuous functions in classical topological spaces via neutrosophic topological spaces. Smarandache [36-39], introduced the concept of NeutroAlgebra and AntiAlgebra thereby laying the foundation for further studies in algebraic structures like groups, rings etc. He used the terms Neutrosophication and Antisophication to generalise the concepts in classical algebra to NeutroAlgebra and AntiAlgebra by defining terms like NeutroAxiom, AntiAxiom, NeutroTheorem, AntiTheorem, NeutroOperation, AntiOperation etc. Agboola *et al*. [40] realized the concept of neutro-algebra and anti-algebra [36-37] by studying them with the help of existing number systems. Further, Agboola [41-42] provided the definition of a neutro-group and neutro-rings. Smarandache *et al*. [43] studied BCK-algebra and extended the study to neutro-BCK-algebra by the application of neutrosophication of the underlying operations of the BCK-algebras. Agboola [44-45] dwelt on finite neutro-groups and finite and infinite neutro-rings. Agboola *et al*. [46-47] introduced anti-groups and anti-rings. Ibrahim *et al*. [48] introduced neutro-vector space and studied certain simple properties of only a particular type of neutro-vector space which they called type 4S. Ibrahim *et al*. [49] defined neutro-hypergroup and anti-hypergroup by neutrosophication and antisophication of the three axioms of a classical hypergroup. Mohammadzadeh *et al*. [50] introduced neutro-nilpotent groups and studied some of their properties and they found the quotient of the group in context and the intersection of two such groups are also of the same group. Al-Tahan *et al*. [51] studied the application of the new algebras to Semigroups by introducing partial order relation in the Neutro-algebras. Smarandache [52], diversified the study of the neutrosophic triplets Truth: <A>; Indeterminate: <Neut A>; Falsity: <Anti A> on various possible studies and analysis of space, event etc. and for any study on any structure, by neutrosophication one can always have the component neutro-structure and by antisophication, one can always have the anti-structure component. Analysis of any structure may be with anything like axiom, theorem, lemma, property, proposition etc. Smarandache [53] extended the study of the neutro and anti-algebras to neutro-geometry and anti-geometry. Rezaei *et al*. [54] extended the study of neutrosophication and antisophication to the study of Semi-hypergroups. Sahin *et al*. [55] defined a neutro-metric space and discussed the basic properties of the neutro-metric, studied various similarities between the neutro-metric and the classical metric and concluded that a neutro-metric is obtainable from every classical metric and also pointed out the variations. Further, Sahin *et al*. [56] introduced the conception of a neutro-topological space and an anti-topological space. The study compared the new topologies with the classical topology and concluded that neutro-topology has a more general structure than the classical topology. They also further concluded that a neutro-topology could be derived from any given classical topology and further that a neutro-topology could also be derived from any given anti-topology. Further, Basumatary *et al*. [57], extended the study on neutro-topological spaces by studying the traits of interior, closure and boundary in neutro-topological spaces and found many interesting results. Again, Basumatary *et al*. [58] defined neutro-bitopological spaces and studied the traits of interior, closure and boundary in the new spaces. Basumatary *et al*. [59] studied the traits of interior, closure and boundary in anti-topological spaces and found that most of the traits that are followed for the aspects in a classical topology are also true in an anti-topology. In a contemporary study made by Witczak [60], wherein study on interior, closure, door spaces was made and two types of continuity of functions was also defined in anti-topological spaces. Basumatary *et al*. [61-62] provided the definitions of neighborhood of a point, base and sub-base in a neutro-topological space and in an anti-topological space and have compared the properties of the aspects to that of the general topological spaces. Basumatary *et al.* [63] established formulae to evaluate the number of neutro-topological spaces based on the number of members in the universal set on which a neutro-topological space is defined.

1.1 Motivation and method of study

The motivation behind this study comes from the fact that in literature, it has been observed that whenever any new set or topology is defined, the primary studies that are carried out in the new structure are studying the properties of interior, closure, boundary, exterior in the new space after which studies are done on continuity of functions in the space and other properties of the new space. After the definition of the neutro-topology has been provided in [56], the properties of interior, closure and boundary in neutro-topological spaces have been studied in [57]. The current study will utilize the notion of the neutro-open and neutro-closed sets defined and used in [57] to define continuity of functions in neutro-topological spaces. And taking advantage of the fact that a neutro-topology could be deduced from any classical topology [56], a new form of continuity called as weakly neutro continuity has also been defined and various underlying properties of continuity of functions are studied in neutro-topological spaces. Further, using the idea of weakly neutro-continuous functions, an attempt has been made to introduce neutro-homeomorphism. Also neutro-open map and neutro-closed map are introduced and they are used to further study the properties of neutro-homeomorphism. The study that has been undertaken has not yet been done by anyone as only a few studies have yet been done in neutro-topological and anti-topological spaces.

2. Preliminaries

Definition 2.1 [56] For a non-void universe X, with a class T of subsets of X, if one or more of {i, ii, iii} below are true, then T becomes a neutro-topology (nu-topology) over X and (X,T) will become a neutro-topological space (*NTTS*):

(i) [ϕ and $X \notin T$ simultaneously] or [$\phi, X \in_I T$]

(ii) For $p_\alpha \in T$, for a finite *n*, $\bigcap_{\alpha=1}^{n} p_\alpha \in T$ and for other $q_\alpha, r_\alpha \in T$, for a finite n, [$\bigcap_{\alpha=1}^{n} q_\alpha \notin T$ or $\bigcap_{\alpha=1}^{n} r_\alpha \in_I T$]

(iii) For $p_\alpha \in T$, $\bigcup_{\alpha\in I} p_\alpha \in T$, I being an arbitrary index set, and for other $q_\alpha, r_\alpha \in T$, [$\bigcup_{\alpha\in I} q_\alpha \notin T$ or, $\bigcup_{\alpha\in I} r_\alpha \in_I T$]

Remark 2.1 [56] The symbols "$=_I$" and "$\in_I$" are used to denote respectively circumstances when "equal to" and "belongs to" are not sure or not properly defined.

Theorem 2.1 [56] If T, a class of subsets of X becomes a topology over X, then $T \setminus \phi$ and $T \setminus X$ are n-topologies on X.

Theorem 2.2 [56] Finite union of n-topologies is again a n-topology.

Definition 2.2 [57] If (X,T) is a $NTTS$, then the components of T are termed neutro-open (nu-open) sets and complements of these nu-open sets are termed neutro-closed (nu-closed) sets.

Definition 2.3 [57] If (X,T) is a $NTTS$ over X and $O \subseteq X$, then the neutro-interior (nu-interior) of O is the join of every nu-open subsets of A and is identified by $NuInt(O)$. That is, $NuInt(O) = \bigcup\{Q_i : Q_i \subseteq O$ and each Q_i is nu-open$\}$.

Theorem 2.3 [57] If A is nu-open the $NuInt(O) = O.$ The result is however not always true the other way around in general.

Theorem 2.4 [57] If (X,T) is a $NTTS$ over X and $O, Q \subseteq X$ then:

(i) $NuInt(O) \subseteq O$

(ii) $NuInt(X) \subseteq X; NuInt(\phi) = \phi$

(iii) $NuInt(NuInt(O)) = NuInt(O)$

(iv) $O \subseteq Q \Rightarrow NuInt(O) \subseteq NuInt(Q)$

Definition 2.4 [57] If (X,T) is a $NTTS$ over X and $C \subseteq X$, then the neutro-closure (nu-closure) of C is the meet of the nu-closed sets that contains C and will be denoted by $NuCl(C)$. Thus, $NuCl(C) = \bigcap\{D_i : C \subseteq D_i$ and each D_i is nu-closed$\}$.

Theorem 2.5 [57] If C is nu-closed then $NuCl(C) = C$. The result is not true the other way around in general.

Theorem 2.6 [57] If (X,T) is a $NTTS$ over X and $C, D \subseteq X$ then the following are true:

(i) $C \subseteq NuCl(C)$

(ii) $NuCl(NuCl(C)) = NuCl(C)$

(iii) $C \subseteq D \Rightarrow NuCl(C) \subseteq NuCl(D)$

3. Continuity in neutro-topological spaces

Definition 3.1. For two $NTTSs$ (X,T_X) and (Y,T_Y), a mapping ξ defined between T_X and T_Y will be neutro-continuous (nu-continuous) if $O \in T_Y \Leftrightarrow \xi^{-1}(O) \in T_X$.

Definition 3.2. For the topological spaces (X,T_X) and (Y,T_Y) the spaces $(X,T_X \setminus \Delta)$ and $(Y,T_Y \setminus \Delta)$ where Δ stands for ϕ or X, are neutro-topological spaces on the sets X and Y by theorem 2.1. A function ξ which is continuous in these neutro-topologies will be termed as weakly neutro-continuous (w-nu-continuous).

Remark 3.1. Whenever w-nu-continuity is mentioned, the underlying nu-topology will be either $T \setminus \phi$ or $T \setminus X$, where T is a classical topology. We will denote the nu-topologies in the $NTTSs$ $(X,T_X \setminus \Delta)$ and $(Y,T_Y \setminus \Delta)$ simply by T_Δ in some of our future discussions.

Theorem 3.1. If ξ is nu-continuous then ξ is w-nu-continuous.

Proof: Straight from their definitions.

Remark 3.2. Theorem 3.1 is not always true the other way around because a classical topology cannot be obtained from a nu-topology by the inclusion of the whole set or the null set to the nu-topology.

Theorem 3.2. For two $NTTSs$ (X, T_X) and (Y, T_Y), a mapping ξ defined between T_X and T_Y will be w-nu-continuous iff for each T_Y-nu-closed C, $\xi^{-1}(C)$ is a T_X-nu-closed.

Proof: If the map f is w-nu-continuous and C is any T_Y-nu-closed set then $C^c(=Y \setminus C) \in T_Y$ and ξ being w-nu-continuous $\xi^{-1}(Y \setminus C) \in T_X$. Now, $\xi^{-1}(Y \setminus C) = X \setminus \xi^{-1}(C) \in T_X$-nu-open set and hence, $\xi^{-1}(C)$ is nu-closed in T_X.
Conversely, if $f^{-1}(C)$ is nu-closed in T_Y for every nu-closed set C in T_X, let $D \in T_Y$. Then $Y \setminus D$ is nu-closed in T_Y and as such $\xi^{-1}(Y \setminus D)$ is nu-closed in T_X. Now, $\xi^{-1}(Y \setminus D) = X - \xi^{-1}(D)$ will be a nu-closed set in T_X thereby showing that $\xi^{-1}(D) \in T_X$. Hence as per definition the map ξ is w-nu-continuous.

Theorem 3.3. For two $NTTSs$ (X, T_X) and (Y, T_Y), a mapping ξ defined between T_X and T_Y will be w-nu-continuous if and only if $NuCl(\xi^{-1}(B) \subseteq \xi^{-1}(NuCl(B))$ for any subset B of Y.

Proof: Assume that ξ is w-nu-continuous and assume that B is any nu-closed subset of Y, then $NuCl(B)$ is nu-closed in T_Y and so by theorem 3.2, $\xi^{-1}(NuCl(B))$ is nu-closed in T_X and hence $NuCl[\xi^{-1}(NuCl(B))] = \xi^{-1}(NuCl(B))$ … (1)
Now, $B \subseteq NuCl(B) \Rightarrow \xi^{-1}(B) \subseteq \xi^{-1}(NuCl(B))$
$\Rightarrow NuCl(\xi^{-1}(B)) \subseteq NuCl(\xi^{-1}(NuCl(B)))$, by theorem 2.6 (iii)
$\Rightarrow NuCl(\xi^{-1}(B)) \subseteq \xi^{-1}(NuCl(B))$, by (1)
Conversely, the condition is assumed to be true and let C be any nu-closed set in T_Y. Then $NuCl(C) = C$ and so by the given condition $NuCl[\xi^{-1}(C)] \subseteq \xi^{-1}(NuCl(C)) = \xi^{-1}(C)$
That is, $NuCl[\xi^{-1}(C)] \subseteq \xi^{-1}(C)$.
But, we have $\xi^{-1}(C) \subseteq NuCl[\xi^{-1}(C)]$ in general by theorem 2.6 (i).
Thus, $NuCl[\xi^{-1}(C)] = \xi^{-1}(C)$, thereby showing that $\xi^{-1}(C)$ is nu-closed in T_X and so by theorem 3.3, the function ξ is w-nu-continuous.

Remark 3.4. In theorem 3.3, the function ξ will not be nu-continuous because in a $NTTS$, the nu-closure of a set is not necessarily a nu-closed set (see theorem 2.5).

Theorem 3.4. For two $NTTSs$ (X, T_X) and (Y, T_Y), a mapping ξ defined between T_X and T_Y will be w-nu-continuous iff $\xi(NuCl(C)) \subseteq NuCl(\xi(C))$ for any subset C of X.

Proof: Consider ξ to be w-nu-continuous and $C \subseteq X$ and assume $\xi(C) = B \subseteq Y$. Then by theorem 3.4, we have $NuCl(\xi^{-1}(B)) \subseteq \xi^{-1}(NuCl(B)))$
$\Rightarrow NuCl(\xi^{-1}(\xi(C))) \subseteq \xi^{-1}(NuCl(\xi(C)))$
$\Rightarrow NuCl(C) \subseteq \xi^{-1}(NuCl(\xi(C)))$, since ξ is w-nu-continuous $\xi^{-1}(\xi(C)) = C$
$\Rightarrow \xi[NuCl(C)] \subseteq \xi[\xi^{-1}(NuCl(\xi(C)))]$
$\Rightarrow \xi(NuCl(C)) \subseteq NuCl(\xi(C))$
Conversely, if the condition is true then let B be any arbitrary nu-closed set in T_Y, then $\xi^{-1}(B) \subseteq X$ and hence by the condition, we have:
$\xi(NuCl(\xi^{-1}(B))) \subseteq NuCl(\xi(\xi^{-1}(B))) \subseteq \xi(\xi^{-1}(NuCl(B))) = \xi(\xi^{-1}(B))$, since B is nu-closed.
$\Rightarrow NuCl(\xi^{-1}(B)) \subseteq \xi^{-1}(B)$

But $\xi^{-1}(B) \subseteq NuCl(\xi^{-1}(B))$, by theorem 2.6 (i).
Hence, $NuCl(\xi^{-1}(B)) = \xi^{-1}(B)$ thereby showing that $\xi^{-1}(B)$ is nu-closed in T_X and so by theorem 3.2, the map ξ is w-nu-continuous.

Remark 3.5. In theorem 3.4, the function ξ will not be nu-continuous because in a $NTTS$, the nu-closure of a set being equal to the set need not necessarily imply that the set is nu-closed (see theorem 2.5).

Theorem 3.5. For two $NTTSs$ (X,T_X) and (Y,T_Y), a mapping ξ defined between T_X and T_Y will be w-nu-continuous iff $\xi^{-1}(NuInt(A)) \subseteq NuInt(\xi^{-1}(A))$ for any subset A of Y.

Proof: Let ξ be w-nu-continuous with $A \subseteq Y$. Then $NuInt(A) \in T_Y$ and so $\xi^{-1}(NuInt(A)) \in T_X$.
Hence $NuInt(\xi^{-1}(NuInt(A))) = \xi^{-1}(NuInt(A))$.
Now $NuInt(A) \subseteq A \Rightarrow \xi^{-1}(NuInt(A)) \subseteq \xi^{-1}(A)$
$\Rightarrow NuInt(\xi^{-1}(NuInt(A))) \subseteq NuInt(\xi^{-1}(A))$
$\Rightarrow \xi^{-1}(NuInt(A)) \subseteq NuInt(\xi^{-1}(A))$, since $\xi^{-1}(NuInt(A)) \in T_X$.
Conversely, assume that the condition is true and let $B \in T_2$ so that $NuInt(B) = B$, then by the given condition we have $\xi^{-1}(NuInt(B)) \subseteq NuInt(\xi^{-1}(B))$ which gives $\xi^{-1}(B) \subseteq NuInt(\xi^{-1}(B))$. But $NuInt(\xi^{-1}(B)) \subseteq \xi^{-1}(B)$ by theorem 2.4 (i) and as such we must have $NuInt(\xi^{-1}(B)) = \xi^{-1}(B)$ thereby showing that $\xi^{-1}(B) \in T_X$ and hence ξ is w-nu-continuous.

Remark 3.6. In theorem 3.5 the function ξ will not be nu-continuous because in a $NTTS$, the nu-interior of a set is not necessarily nu-open (see theorem 2.3).

Theorem 3.6. For three $NTTSs$ $(X,T_X),(Y,T_Y),$ and (Z,T_Z), if the maps ξ from T_X to T_Y and η from T_Y to T_Z are nu-continuous, then the map from T_X to T_Z given by $\eta \circ \xi : (X,T_X) \to (Z,T_Z)$ is also nu-continuous.

Proof: Assume that C be a nu-open set in T_Z, then $\eta^{-1}(C) \in T_Y$ and $\xi^{-1}(\eta^{-1}(C)) \in T_X$. But $\xi^{-1}(\eta^{-1}(C)) = (\xi^{-1} \circ \eta^{-1})(C) = (\eta \circ \xi)^{-1}(C)$. Thus, $(\eta \circ \xi)^{-1}(C) \in T_X$ whenever $C \in T_Z$ and so the map $(\eta \circ \xi)$ is nu-continuous.

Theorem 3.7. If (X,T_1) and (Y,T_2) are two $NTTSs$ and if $\{x\}$ is a singleton subset of X, then the map $\xi : (X,T_X) \to (Y,T_Y)$ is nu-continuous at $x \in X$.

Proof: Let $\xi(x) \in B \subseteq Y \Rightarrow x \in \xi^{-1}(B) \Rightarrow \{x\} \in \xi^{-1}(B) \Rightarrow \xi$ is nu-continuous at $x \in X$.

Theorem 3.8. For a $NTTS$ (X,T), the map $\xi : X \to X$ defined by $\xi(x) = x$ for every $x \in X$ is nu-continuous.

Proof: Let $B \in T \Rightarrow B \subseteq X$. Now, $\xi(x) = x \quad \forall x \in X$ and $B \subseteq X$
$\Rightarrow \xi^{-1}(B) = \{x \in X : \xi(x) \in B\} \Rightarrow \xi^{-1}(B) = \{x \in X : x \in B\} \Rightarrow \xi^{-1}(B) = \{x\}$
$\Rightarrow \xi$ is nu-continuous.

Definition 3.3. A map ξ between two $NTTSs$ (X,T_X) and (Y,T_Y) is called a neutro-open (nu-open) map if ξ images of nu-open sets in T_X are nu-open sets in T_Y. The map

$\xi:(X,T_X)\to(Y,T_Y)$ will be called a neutro-closed (nu-closed) map if the ξ images of T_X nu-closed sets are T_Y nu-closed sets.

Definition 3.4. If (X,T_X) and (Y,T_Y) are two $NTTSs$, then a map $\xi:(X,T_X)\to(Y,T_Y)$ will be called a neutro-homeomorphism (nu-homeomorphism) iff:

(i) ξ is one-one and onto;

(ii) $\xi: X\to Y$ is w-nu-continuous; and

(iii) $\xi^{-1}: Y\to X$ is w-nu-continuous.

If such a map ξ exists between the $NTTSs$ (X,T_X) and (Y,T_Y), then the two $NTTSs$ will be neutro-homomorphic to each other.

Theorem 3.9. For two $NTTSs$ (X,T_X) and (Y,T_Y), if ξ is one-one and onto mapping of X to Y, then ξ is a nu-homeomorphism iff ξ is w-nu-continuous and a nu-open map.

Proof: Assume that ξ is a nu-homeomorphism and let $\xi^{-1}=\eta$ and $\eta^{-1}=\xi$. Since ξ is one-one and onto, so η will also be one-one and onto. Now, if $O\in T_X$, then $\eta^{-1}(O)\in T_Y$. But since $\eta^{-1}=\xi$ so $\eta^{-1}(O)=\xi(O)\in T_Y$. Thus $O\in T_1\Rightarrow \xi(O)\in T_2$ and hence it follows that ξ is a nu-open map and since ξ is a nu-homeomorphism, so it is w-nu-continuous.

Conversely, let ξ be w-nu-continuous and a nu-open map. Also by condition ξ is one-one and onto, so it suffices to prove that $\xi^{-1}=\eta$ is w-nu-continuous. Let $O\in T_X$, then $\xi(O)\in T_Y$ since ξ is nu-open. That is $\eta^{-1}(O)\in T_Y$ thereby showing that $\eta=\xi^{-1}$ is w-nu-continuous. Hence the map ξ is a nu-homeomorphism.

Theorem 3.10. For two $NTTSs$ (X,T_X) and (Y,T_Y), if ξ is one-one and onto mapping of X to Y, then ξ is a nu-homeomorphism iff ξ is w-nu-continuous and a nu-closed map.

Proof: Let ξ be a nu-homeomorphism and let C be any nu-closed set in T_X then $X\setminus C\in T_X$. Since $\eta=\xi^{-1}$ is w-nu-continuous, it follows that $\eta^{-1}(X \setminus C)\in T_Y$. Now, $\eta^{-1}(X\setminus C)=Y\setminus \eta^{-1}(C)$ and hence $\eta^{-1}(C)$ is nu-open in T_Y and as such $\eta^{-1}(C)$ is nu-closed in T_Y. That is, $\eta^{-1}(C)=\xi(C)$ is nu-closed in T_Y. Hence ξ is w-nu-continuous and a nu-closed map.

Conversely, let the conditions hold and let O be any nu-open set in T_X, then $X\setminus O$ is nu-closed set and ξ is a nu-closed map, so $\xi(X\setminus O)=\xi(X\setminus O)=Y\setminus\eta^{-1}(O)$ is a nu-closed set in T_Y which implies that $\eta^{-1}(O)\in T_Y$. Thus the image of every nu-open set in T_X under the function η is nu-open in T_Y. Thus, $\eta=\xi^{-1}$ is w-nu-continuous and hence ξ is a nu-homeomorphism.

Theorem 3.11. For two $NTTSs$ (X,T_X) and (Y,T_Y), if a mapping ξ from T_X to T_Y is one-one onto and w-nu-continuous, then ξ is a nu-homeomorphism if ξ is nu-open or nu-closed.

Proof: We assume that ξ is one-one onto and w-nu-continuous and also that ξ is either nu-open or nu-closed. We have to show that ξ^{-1} is w-nu-continuous. By theorem 3.4, it suffices to verify that $\xi^{-1}(NuCl(B))\subseteq NuCl(\xi^{-1}(B))$ for any subset B of Y.

Now, $B\subseteq Y\Rightarrow NuCl(\xi^{-1}(B))\subseteq X$ and is nu-closed. Also since ξ is given to be nu-closed, we have: $\xi(NuCl(\xi^{-1}(B)))=NuCl(\xi(NuCl(\xi^{-1}(B))))\ldots$ (1)

Also, $\xi^{-1}(B)\subseteq NuCl(\xi^{-1}(B))\Rightarrow \xi(\xi^{-1}(B))\subseteq \xi(NuCl(\xi^{-1}(B)))$

$\Rightarrow NuCl(\xi(\xi^{-1}(B))) \subseteq NuCl(\xi(NuCl(\xi^{-1}(B)))) = \xi(NuCl(\xi^{-1}(B)))$ by (1)
$\Rightarrow NuCl(B) \subseteq \xi(NuCl(\xi^{-1}(B))) \Rightarrow \xi^{-1}(NuCl(B)) \subseteq NuCl(\xi^{-1}(B))$
Hence by theorem 3.3 the map ξ^{-1} is w-nu-continuous and hence ξ is a nu-homeomorphism.

Theorem 3.12. For two $NTTSs$ $(X, T_{X,\Delta})$ and $(Y, T_{Y,\Delta})$, a function ξ between the two spaces is nu-open iff $\xi(NuInt(A)) \subseteq NuInt(\xi(A))$ for any $A \subseteq X$.

Proof: Assume ξ to be nu-open and $A \subseteq X$ then $\xi(NuInt(A))$ is nu-open since $NuInt(A) \in T_{X,\Delta}$. Now $NuInt(A) \subseteq A \Rightarrow \xi(NuInt(A)) \subseteq \xi(A)$.
Again $\xi(NuInt(A)) \in T_{Y,\Delta}$, so $NuInt(\xi(NuInt(A))) = \xi(NuInt(A))$ … (1)
Also, $\xi(NuInt(A)) \subseteq \xi(A) \Rightarrow NuInt(\xi(NuInt(A))) \subseteq NuInt(\xi(A))$
$\Rightarrow \xi(nInt(A)) \subseteq nInt(\xi(A))$ by (1)
Conversely, let the condition be true and let $O \in T_{X,\Delta}$ so that $NuInt(O) = O$. Then $\xi(O) = \xi(NuInt(O)) \subseteq NuInt(\xi(O))$, by the given condition. But we have by theorem 2.4 (i) $NuInt(\xi(O)) \subseteq \xi(O)$. Thus, we have $NuInt(\xi(O)) = \xi(O)$ thereby showing that $\xi(O) \in T_{Y,\Delta}$. Thus ξ is nu-open.

Remark 3.7. In the above theorem T_X and T_Y are two different nu-topologies both having the property of T_Δ (see remark 3.1) and so have been denoted by $(X, T_{X,\Delta})$ and $(Y, T_{Y,\Delta})$.

Theorem 3.13. For two $NTTSs$ $(X, T_{X,\Delta})$ and $(Y, T_{Y,\Delta})$, a function ξ between the two spaces is nu-closed iff $NuCl(\xi(C)) \subseteq \xi(NuCl(C))$ for any $C \subseteq X$.

Proof: Let ξ be nu-closed and $C \subseteq X$, then $NuCl(C)$ is nu-closed in $T_{X,\Delta}$ and since ξ is nu-closed $\xi(NuCl(C))$ is nu-closed in $T_{Y,\Delta}$ and thus $NuCl(\xi(NuCl(C))) = \xi(NuCl(C))$ … (1)
Again,
$C \subseteq NuCl(C) \Rightarrow \xi(C) \subseteq \xi(NuCl(C)) \Rightarrow NuCl(\xi(C)) \subseteq NuCl(\xi(NuCl(C))) = \xi(NuCl(C))$
by (1). That is $NuCl(\xi(C)) \subseteq \xi(NuCl(C))$.
Conversely, let the condition hold and suppose that D is some nu-closed set in $T_{X,\Delta}$ so that $NuCl(D) = D$. Then $\xi(NuCl(D)) = \xi(D)$ … (2)
Now, by the given condition $NuCl(\xi(D)) \subseteq \xi(NuCl(D)) = \xi(D)$ by (2)
That is, $NuCl(\xi(D)) \subseteq \xi(D)$. But by theorem 2.6 (i) we have $\xi(D) \subseteq NuCl(\xi(D))$ as a result of which we must have $NuCl(\xi(D)) = \xi(D)$ thereby showing that $\xi(D)$ is nu-closed in $T_{Y,\Delta}$. Hence ξ is nu-closed.

Theorem 3.14. For two $NTTSs$ (X, T_X) and (Y, T_Y), if the function ξ between the two spaces is one-one onto, then ξ is a nu-homeomorphism iff $NuCl(\xi(C)) = \xi(NuCl(C))$ for every $C \subseteq X$.

Proof: Assume that ξ is a nu-homeomorphism. Then ξ is one-one, onto, and also by theorem 3.11 ξ is nu-closed and w-nu-continuous. Now, if $C \subseteq X$, then by theorem 3.4 $\xi(NuCl(C)) \subseteq NuCl(\xi(C))$ … (1)
Also, $C \subseteq NuCl(C) \Rightarrow \xi(C) \subseteq \xi(NuCl(C)) \Rightarrow NuCl(\xi(C)) \subseteq NuCl(\xi(nCl(C))$ … (2)
Now, ξ is nu-closed and $NuCl(C)$ is nu-closed in T_X and so $\xi(NuCl(C))$ is nu-closed in T_Y and hence $NuCl(\xi(NuCl(C))) = \xi(NuCl(C))$ … (3)

(2) and (3) gives $NuCl(\xi(C)) \subseteq \xi(NuCl(C))$ ... (4)
(1) and (4) gives $\xi(NuCl(C)) = NuCl(\xi(C))$
Conversely, let $\xi(NuCl(C)) = NuCl(\xi(C))$ for every $C \subseteq X$.
Then obviously $\xi(NuCl(C)) \subseteq NuCl(\xi(C))$ which from theorem 3.4 means that the function ξ is w-nu-continuous. Again, if D is any nu-closed set in T_X so that $NuCl(D) = D$ which implies $\xi(NuCl(D)) = \xi(D) \Rightarrow \xi(D) = NuCl(\xi(D))$ by the given condition, thereby showing that $\xi(D)$ is nu-closed in T_Y whenever D is closed in T_X thereby showing that ξ is nu-closed. Thus ξ is nu-closed as well as one-one onto and w-nu-continuous, so ξ is a nu-homeomorphism.

4. Comparative view of neutro-continuity, neutrosophic continuity vis-à-vis classical continuity

In a neutro-topological space the interior of a set is not open in general and the closure of a set is also not generally closed. However, if we choose the neutro-topological space (X, T_Δ), where T_Δ is the neutro-topology obtained by either the exclusion of the whole set or the null set from the topology T, then we can have the interior of any subset open and the closure of any subset closed by choosing the exclusion of the null set or the whole set in the neutro-topology appropriately. Below we list some basic properties that are true in the three topological spaces:

Classical Continuity	Neutrosophic Continuity	Nu-continuity and w-nu-continuity
If $(X,T_X),(Y,T_Y)$, and (Z,T_Z) are three topological spaces and $\xi: X \to Y$ and $\eta: Y \to Z$ are continuous functions then $\eta \circ \xi: X \to Z$ is a continuous function.	True. Theorem 3.1 [35].	True. Theorem 3.6.
A mapping ξ from a space X into another space Y is continuous if and only if $\xi(\overline{A}) \subset \overline{\xi(A)}$ for every $A \subset X$.	True. Theorem 3.3 [35].	True only for w-nu-continuity. Theorem 3.4 and remark 3.5.
A mapping ξ from a space X into another space Y is continuous if and only if $\overline{\xi^{-1}(B)} \subset \xi^{-1}(\overline{B})$ for every $B \subset Y$.	True. Theorem 3.4 [35].	True only for w-nu-continuity. Theorem 3.3 and remark 3.4.
A mapping from a space X into another space Y is continuous if and only if $\xi^{-1}[B^\circ] \subset [\xi^{-1}(B)]^\circ$ for every $B \subset Y$.	True. Theorem 3.6 [35].	True only for w-nu-continuity. Theorem 3.5 and remark 3.6.
If a mapping $\xi: X \to Y$ between the spaces X and Y is one-one onto then ξ is a homeomorphism if and only if ξ is continuous and closed.	True. Theorem 3.12 [35]	True for w-nu-homeomorphism. Theorem 3.10.

4. Conclusions

In this article, we have introduced the definition of neutro-continuous functions in neutro-topological spaces with the use of the open sets called neutro-open sets. Taking advantage of the fact that a neutro-topology could be deduced from any classical topology with the exclusion of the empty set or the whole set, we have defined a new form of continuity of functions in such neutro-topological spaces and named it weakly neutro-continuity. In a general neutro-topology the union and intersection of the neutro-open sets may or may not be neutro-open. However, in the neutro-topology which is obtained by the simple exclusion of the empty set from a topology, the unions of the neutro-open sets are neutro-open and in the neutro-topology obtained from a topology by the exclusion of the whole set, the intersection of the neutro-open sets are neutro-open. It has been assumed that whenever properties of interior of neutro-open sets are involved, the exclusion of the whole set is considered in the neutro-topology and whenever closure properties are involved the exclusion of the empty set should be considered. Thus, we have found the results involving the interior and closure of subsets valid via the weakly neutro-continuity of the functions. It has also been observed that whenever a function is weakly neutro-continuous, it is not always neutro-continuous because in a neutro-topology the union and intersection of the open and closed sets involved in the very definition of the neutro-topology may not be included in the collection that forms the neutro-topology.

4.1 Limitations

In our analysis we have found that most of the properties of continuous functions are true only in the case of weakly neutro-continuity, which has been defined in a special type of a neutro-topology. Neutro-homeomorphism has also been defined on the basis of weakly neutro-continuity and will not be generally meaningful if defined with the help of neutro-continuity as most of the properties that have been established might not be true in a general neutro-topological space.

4.2 Scope for further studies in the field

Further, one could investigate the continuity of functions with regard to the neutro-neighborhood, neutro-base, neutro-sub-base, neutro-relative topology and via other aspects of study of continuity of functions. The current study can be extended to study uniform continuity that will help study connectedness and compactness in neutro-topological spaces.

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