A STUDY ON THE PROPERTIES OF NEUTROTOPOLOGICAL AND ANTITOPOLOGICAL SPACES

A THESIS
SUBMITTED TO BODOLAND UNIVERSITY IN PARTIAL
FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY
IN
MATHEMATICS

SUBMITTED BY
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Registration No.: **MAT00363** of 2021-22

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2024

Summary and Conclusion

8.1   Summary and Conclusion

When the definition of neutro-topological space (*N-TS*) and anti-topological space (*A-TS*) was observed for the first time, a curiousness was felt by observing the nature of the definition of the two new spaces. The very definitions and the few results that were added to the research article containing the definitions, led to the question how the new spaces will behave with respect to the preliminary aspects of studies that are usually undertaken when any new kind of topological space is defined. Thus, it was felt necessary to study the properties of interior, exterior, closure, and boundary in the two new spaces. It was also felt necessary to observe how properties of continuous functions would be like in the two new spaces. More importantly, separation axioms in these new studies were felt necessary to be studied. Further, neutro-bi-topological space (*N-B-T-S*) was also felt necessary to be studied and hence the aspects of interior, closure, and boundary are defined in *N-B-TS* and their properties are analyzed. Further, since multisets have been studied with different types of topological spaces, it was felt necessary that the same could be done with the two new types of spaces.

Thus, this thesis is about the study of some basic properties of *N-TS*, *N-B-TS*, and *A-TS*. In chapter 1, some introduction has been provided with the emphasis on the evolution of the study of topological spaces over the decades. It has been shown how the concept of the open set has evolved over time thereby generating new definitions of topologies leading to the definition of the *N-TS* and *A-TS* in 2021. In order to study the properties of the new spaces some definitions that are prevalent in general topological spaces have also been provided in the first chapter.

In Chapter 2, after defining the notions of neutro-interior, neutro-exterior, neutro-closure, and neutro-boundary in neutro-topological spaces, all the properties that are classically valid in general topological spaces, for the aspects of interior, exterior, closure and boundary have been analyzed and all the properties that are correspondingly valid in neutro-topological space have been proved. A few deviations have been observed in some of the properties and the reasons for the deviations have also been provided in the form of remarks and proper counter examples have also been provided

to justify the deviations. Further, subspace of a neutro-topological space has also been defined and some relevant properties have been studied in the subspace.

Chapter 3 introduces the notion of a neutro-bi-topological space for the first time. Correspondingly, the notions of neutro-bi-interior, neutro-bi-closure, and neutro-bi-boundary have been defined and the properties are analyzed with respect to the new definitions. Even though, neutro-bi-exterior was defined, further properties could not be studied because of the absence of a direction between the neutro-bi-exterior and the neutro-bi-interior. However, a new concept of neutro-pseudo-exterior has been defined through which many properties could be established. Further, a weaker type of neutro-bi-topological space has been introduced, through neutro-quasi open sets and neutro-quasi closed sets and correspondingly the concepts of neutro-quasi-interior, neutro-quasi closure, and neutro-quasi boundary have been introduced and the corresponding properties analyzed.

Chapter 4 uses the definitions of anti-open sets and anti-closed sets to define anti-interior, anti-exterior, anti-closure, and anti-boundary in anti-topological spaces and the properties of the corresponding aspects in general topological spaces are analyzed and surprisingly it has been found that most of the properties are also true in the anti-topological spaces with a few exceptions. The deviations in certain properties have been justified with the reasons in the form of remarks and proper counter examples to justify the reasons for the variations.

Chapter 5 deals with continuity of functions in neutro-topological spaces and anti-topological spaces. The concept of the openness of the inverse image of open sets has been used to define the continuity in both the spaces. However, in neutro-topological spaces, the openness of inverse image of closed set has also been used. Further, taking advantage of the fact that a *N-TS* could be deduced from any topological space, the concept of weakly neutro-continuity has been defined and most of the properties of continuous functions in general topological spaces have been found to be valid in such neutro-topological space with the weakly neutro-continuity. Further, using the concept of the weakly neutro-continuous functions, neutro-homeomorphism has been defined and many results of neutro-homeomorphism have been established. In the case of continuity in anti-topological spaces, only a few results could be established

because of the nature of the space and a concept like weakly anti-continuous could not be visualized and hence no other extended study could be made.

Chapter 6 extends the study of separation axioms of spaces in topological spaces to *N-TS* and *A-TS*. The axioms of T_0, T_1, T_2, regularity, T_3, normality, and T_4 have been defined in both the *N-TS* and *A-TS* and various hereditary properties have been studied with the types of open and closed sets in the two spaces. And it has been found that most of the hereditary properties are preserved in both the spaces with the exception of a few and the exceptions are due to the nature of definition of the two new spaces. And because of the fact that a *N-TS* could be deduced from any topological space, for every axiom of separation in a topological space, we have a corresponding neutro-topological space with the same axiom of separation. Moreover, for every anti-topological space equipped with a certain axiom of separation, we have a corresponding neutro-topological space with the same axiom.

Chapter 7 uses the concept of multisets to define *M-N-TS*, *M-N-B-TS*, *M-A-TS*. Further, the aspects of multi-neutro (*MN*)-interior, *MN*-exterior, *MN*-closure and *MN*-boundary are defined and the results of these aspects already established in *N-TS* are extended to the *M-N-TS*. Further, the aspects of *MN-Bi-Interior*, *MN-Bi-Closure*, *MN-Bi-Boundary* are defined and the results of these aspects that are established in *N-B-TS* are extended to the *M-N-BTS*. Further, with the help of multisets, the aspects of multi-anti (*MA*)-interior, *MA*-exterior, *MA*-closure, and *MA*-boundary are defined and the results of these aspects already established in *A-TS* are studied in the *M-A-TS*.

8.2   Future Scope

There are many aspects of topological spaces that can be studied on the basis of the study that we have undertaken in *NTS* and *ATS*. Some of the following aspects may be considered for further studies:

- Connectedness in neutro-topological and anti-topological spaces
- Compactness in neutro-topological and anti-topological spaces
- Convergence in neutro-topological and anti-topological spaces
- Uniform continuity in neutro-topological and anti-topological spaces
- Finite product of neutro-topological and anti-topological spaces