A STUDY ON THE PROPERTIES OF NEUTROTOPOLOGICAL AND ANTITOPOLOGICAL SPACES

A THESIS
SUBMITTED TO BODOLAND UNIVERSITY IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY
IN
MATHEMATICS

SUBMITTED BY
JEEVAN KRISHNA KHAKLARY

Registration No.: **MAT00363** of 2021-22

UNDER THE SUPERVISION OF
DR. BHIMRAJ BASUMATARY

DEPARTMENT OF MATHEMATICAL SCIENCES

FACULTY OF SCIENCE & TECHNOLOGY
BODOLAND UNIVERSITY, KOKRAJHAR
ASSAM - 783370, INDIA

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DECLARATION

I hereby declare that I have carried out the current research work entitled "**A Study on the properties of NeutroTopological and AntiTopological Spaces**" under the supervision and guidance of Dr. Bhimraj Basumatary, Assistant Professor, Department of Mathematical Sciences, Bodoland University, Kokrajhar, Assam, India. The thesis has been submitted to Bodoland University for the award of the degree "Doctor of Philosophy" in the Faculty of Science & Technology.

I further declare that the results and analyses presented in the thesis represent my work in the original form that has not been submitted previously for a degree or diploma to any university or institute of higher education.

Jeevan Krishna Khaklary

(Jeevan Krishna Khaklary)
Research Scholar
Deptt. of Mathematical Sciences
Bodoland University, Kokrajhar

Date: 26/02/2025

DEPARTMENT OF MATHEMATICAL SCIENCES

Dr. Bhimraj Basumatary

Assistant Professor

Bodoland University

Kokrajhar, Assam – 783370, India

+91-9508908682

Email: brbasumatary14@gmail.com

CERTIFICATE

This is to certify that the thesis titled "**A Study on the Properties of Neutro-Topological and Anti-Topological Spaces**" has been submitted by **Mr. Jeevan Krishna Khaklary** for the award of the Degree of Doctor of Philosophy in Mathematics, to Bodoland University, Kokrajhar, Assam, India, as a record of bonafide research work carried out by him under my supervision in the Department of Mathematical Sciences, Bodoland University, Kokrajhar.

The thesis satisfies the requirements of the regulation relating to the degree. Also, considerable parts of the thesis have been published in national and international journals. The work reported in the thesis is original and has not been submitted to any other university or institute for the award of any degree or diploma.

(Dr. Bhimraj Basumatary)

Assistant Professor

Supervisor

Department of Mathematical Sciences

Bodoland University, Kokrajhar

Date: 26/02/2025

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Jeevan Krishna Khaklary

Jeevan Krishna Khaklary

Date: 26/02/2025

Research Scholar

List of Abbreviations

A-C	*Anti-Closed*
A-CS	*Anti-Closed Set*
A-CSs	*Anti-Closed Sets*
A-O	*Anti-Open*
A-OS	*Anti-Open Set*
A-OSs	*Anti-Open Sets*
A-T	*Anti-Topology*
A-TS	*Anti-Topological Space*
A-TSs	*Anti-Topological Spaces*
B-TS	*Bi-topological Space*
FS	*Fuzzy Set*
FT	*Fuzzy Topology*
FTS	*Fuzzy Topological Space*
GTS	*General Topological Space*
iff	*If and only if*
IFS	*Intuitionistic Fuzzy Set*
IFTS	*Intuitionistic Fuzzy Topological Space*

M-A	*Multi-Anti*
M-A-Bd	*Multi-Anti-Boundary*
M-A-C	*Multi-Anti-Closed*
M-A-Cl	*Multi-Anti-Closure*
M-A-Ext	*Multi-Anti-Exterior*
M-A-Int	*Multi-Anti-Interior*
M-A-O	*Multi-Anti-Open*
M-A-T	*Multi-Antti-Topology*
M-A-TS	*Multi-Anti-Topological Space*
M-B-TS	*Multi-Bi-topological Space*
M-N	*Multi-Neutro*
M-N-Bd	*Multi-Neutro-Boundary*
M-N-B-TS	*Multi-Neutro-Bi-Topological Space*
M-N-C	*Multi-Neutro-Closed*
M-N-Cl	*Multi-Neutro-Closure*
M-N-Ext	*Multi-Neutro-Exterior*
M-N-Int	*Multi-Neutro-Interior*
M-N-O	*Multi-Neutro-Open*
M-N-T	*Multi-Neutro-Topology*
M-N-TS	*Multi-Neutro-Topological Space*
mset	*Multiset*
M-TS	*Multi-Topological Space*
N-B-TS	*Neutro-Bi-topological Space*
N-C	*Neutro-Closed*
N-CS	*Neutro-Closed Set*
N-CSs	*Neutro-Closed Sets*

nhd	*Neighbourhood*
N-O	*Neutro-Open*
N-OS	*Neutro-Open Set*
N-OSs	*Neutro-Open Sets*
N-PO	*Neutro-Pseudo Open*
N-QC	*Neutro-Quasi Closed*
N-QO	*Neutro-Quasi Open*
NS	*Neutrosophic Set*
N-T	*Neutro-Topology*
N-Ts	*Neutro-Topologies*
NTS	*Neutrosophic Topological Space*
N-TS	*Neutro-Topological Space*
N-TSs	*Neutro-Topological Spaces*
Nu	*Neutro*
Nu-nhd	*Neutro-Neighborhood*
Submset	*Sub Multi-set*
w.r.t.	*with respect to*

List of Notations

$c\mathcal{A}$	Complement of the set $\mathcal{A}$
$\mathcal{A}^{Anti-bd}$	Anti-Boundary of the set $\mathcal{A}$
$\mathcal{A}^{Anti-cl}$	Anti-Closure of the set $\mathcal{A}$
$\mathcal{A}^{Anti-ext}$	Anti-Exterior of the set $\mathcal{A}$
$\mathcal{A}^{Anti-int}$	Anti-Interior of the set $\mathcal{A}$
$\mathcal{A}_{\mathcal{Y}}^{Anti-cl}$	Anti-Closure of $\mathcal{A}$ relative to the set $\mathcal{Y}$
$\mathcal{A}_{\mathcal{Y}}^{Anti-int}$	Anti-Interior of $\mathcal{A}$ relative to the set $\mathcal{Y}$
$\mathcal{A}^{MA-Bd}$	Multi-Anti-Boundary of $\mathcal{A}$
$\mathcal{A}^{MA-Cl}$	Multi-Anti-Closure of $\mathcal{A}$
$\mathcal{A}^{MA-Ext}$	Multi-Anti-Exterior of $\mathcal{A}$
$\mathcal{A}^{MA-Int}$	Multi-Anti-Interior of $\mathcal{A}$
$\mathcal{A}^{MN-Bd}$	Multi-Neutro-Boundary of $\mathcal{A}$
$\mathcal{A}^{MN-Cl}$	Multi-Neutro-Closure of $\mathcal{A}$
$\mathcal{A}^{MN-Ext}$	Multi-Neutro-Exterior of $\mathcal{A}$
$\mathcal{A}^{MN-Int}$	Multi-Neutro-Interior of $\mathcal{A}$
$\mathcal{A}^{NQ-cl}$	Neutro-Quasi-Closure of the set $\mathcal{A}$
$\mathcal{A}^{NQ-int}$	Neutro-Quasi-Interior of the set $\mathcal{A}$
$\mathcal{A}^{Nu-bd}$	Neutro-Boundary of the set $\mathcal{A}$
$\mathcal{A}^{Nu-cl}$	Neutro-Closure of the set $\mathcal{A}$

$\mathcal{A}^{Nu-ext}$	Neutro-Exterior of the set $\mathcal{A}$
$\mathcal{A}^{Nu-int}$	Neutro-Interior of the set $\mathcal{A}$
$\mathcal{A}_{MN}^{\mathcal{T}_1-Cl}$	Multi-Neutro-Closure with respect to $\mathcal{T}_1$
$\mathcal{A}_{MN}^{\mathcal{T}_1-Int}$	Multi-Neutro-Interior with respect to $\mathcal{T}_1$
$\mathcal{A}_{MN}^{\mathcal{T}_{12}-Bd}$	Multi-Neutro-Boundary
$\mathcal{A}_{MN}^{\mathcal{T}_{12}-Cl}$	Multi-Neutro-Bi-Closure: Multi-Neutro-Closure with respect to $\mathcal{T}_1$ of the Multi-Neutro-Closure of $\mathcal{A}$ with respect to $\mathcal{T}_2$
$\mathcal{A}_{MN}^{\mathcal{T}_{12}-Int}$	Multi-Neutro-Bi-Interior: Multi-Neutro-Interior with respect to $\mathcal{T}_1$ of the Multi-Neutro-Interior of $\mathcal{A}$ with respect to $\mathcal{T}_2$
$\mathcal{A}^{\mathcal{T}_1-Nbd}$	Neutro-Boundary with respect to $\mathcal{T}_1$
$\mathcal{A}^{\mathcal{T}_1-Ncl}$	Neutro-Closure with respect to $\mathcal{T}_1$
$\mathcal{A}^{\mathcal{T}_1-Next}$	Neutro-Exterior with respect to $\mathcal{T}_1$
$\mathcal{A}^{\mathcal{T}_1-Nint}$	Neutro-Interior with respect to $\mathcal{T}_1$
$\mathcal{A}^{\mathcal{T}_{12}-Nbd}$	Neutro-Bi-Boundary: Neutro-boundary with respect to $\mathcal{T}_1$ of the neutro-boundary of $\mathcal{A}$ with respect to $\mathcal{T}_2$
$\mathcal{A}^{\mathcal{T}_{12}-Ncl}$	Neutro-Bi-Closure: Neutro-closure with respect to $\mathcal{T}_1$ of the neutro-closure of $\mathcal{A}$ with respect to $\mathcal{T}_2$
$\mathcal{A}^{\mathcal{T}_{12}-Next}$	Neutro-Bi-Exterior: Neutro-exterior with respect to $\mathcal{T}_1$ of the neutro-exterior of $\mathcal{A}$ with respect to $\mathcal{T}_2$
$\mathcal{A}^{\mathcal{T}_{12}-Nint}$	Neutro-Bi-interior: Neutro-Interior with respect to $\mathcal{T}_1$ of the neutro-interior of $\mathcal{A}$ with respect to $\mathcal{T}_2$
$\mathcal{A}^{\mathcal{T}_{12}^{p}-Next}$	Neutro-Pseudo-Exterior with respect to $\mathcal{T}_{12}$
T_0^A	Anti-T_0
T_1^A	Anti-T_1
T_2^A	Anti-T_2

T_3^A	Anti-T_3
T_4^A	Anti-T_4
T_0^N	Neutro-T_0
T_1^N	Neutro-T_1
T_2^N	Neutro-T_2
T_3^N	Neutro-T_3
T_4^N	Neutro-T_4