Abstract of the Thesis

Introduction:

Study on topological spaces have been done on the basis of the classical Cantorian set where all the members are well defined. But in many real-life situations, many objects, items, notions, etc. intended to be defined in the form of a set happens to be not properly defined all the time. In many such situations the Cantorian set fails to encompass the notion of a set. Moving away from the Cantorian set, the first attempt to define a new form of a set which would deal with the shortfalls of the Cantorian set, was done by Zadeh [22] in 1965 when he defined a fuzzy set. Unlike in the Cantorian set where a member either belonged to the set or not; in the fuzzy set, every member has a membership grade and not restricted to being a complete member of a non-member. In other words, membership of the element can be partial in a fuzzy set. With the introduction of the fuzzy set much of attention was deviated to the study of topological aspects via the fuzzy set and as such, three years later, in 1968 Chang [6] defined fuzzy topological space with fuzzy set being the basis of the study. In this study he defined fuzzy neighbourhood of a point and introduced the study of continuity of functions in fuzzy topological spaces. Nazaroff [10] defined interior, exterior, closure and boundary in the fuzzy topology and studied the results associated with these topological aspects. Wong [20] studied local countability, separability and local compactness in fuzzy topology. Hutton *et al.* [8] studied the axioms of separation in the fuzzy topology. Yager [21] defined fuzzy multiset and used it as a tool to study the cardinality of fuzzy set. It is noteworthy to point out that the fuzzy set that have been studied this far had the membership grade for all the elements in the fuzzy set and nothing of the sort of non-membership grade was discussed until 1986, when Atanassov [5] defined the intuitionistic fuzzy set. In an intuitionistic fuzzy set, every member has a membership grade and a non-membership grade. Further, intuitionistic fuzzy topological space was brought about by Coker [7] in 1997. Corresponding to the definition of the intuitionistic fuzzy topological spaces, studies on the aspects of interior, closure, boundary, continuity of functions and separation axioms have been done by Coker [7] and various other scholars. However, it was unsure whether the membership grade and non-membership grade of an element complemented each other. This led to the introduction of the neutrosophic logic by Smarandache [14] in 1998. According to this logic, a certain degree of uncertainty always surrounds the study of fuzziness. In other words, the intuitionistic fuzzy set would not provide the accurate study of fuzziness by its membership and non-membership grades. This led to the definition of a neutrosophic set by Smarandache [15] in 2005. The new set has three components namely, truth (membership), indeterminacy, and falsity (non-membership). Thus, the neutrosophic set was a refinement of the intuitionistic fuzzy set because of the inclusion of the indeterminacy grade as a third component of the fuzzy set. Salama *et al.* [13] formalized the notion of the neutrosophic set and provided the definition of neutrosophic topological space based on the neutrosophic set. They also provided the definitions of interior and closure in the neutrosophic topological spaces.

Smarandache [16-19] extended the concept of the neutrosophic logic to define neutro-algebra and anti-algebra. He used the terms neutrosophication and antisophication to generalize concepts in classical algebra to neutro-algebra and anti-algebra. Agboola [1, 2] extended the study on neutro-algebra by defining neutro-group and neutro-ring. Agboola [3, 4] extended the study on the anti-algebras by introducing anti-group and anti-rings. Ibrahim *et al*. [9] introduced neutro-vector space. Sahin *et al.* [11] in 2021 defined neutro-matric space and studied convergence in neutro-metric space by defining a neutro-Cauchy sequence. Further, Sahin *et al*. [12] introduced the concept of a neutro-topological space and anti-topological space on the basis of the ideas of neutrosophication and antisophication used by Smarandache [17]. In the new topological structure even though the neutrosophic concept have been used, the neutrosophic set has not been used. In other words, the study made on the neutro-topology and the anti-topology have been made on the basis of the classical set. In the study made in [12], a few basic results are discussed and results like every classical topology can be converted to a neutro-topology and every anti-topology can be converted to a neutro-topology. On the basis of the literature review this study aims to continue the study of the aspect of interior, exterior, closure, boundary, continuity of functions, separation axioms and multisets with respect to the neutro-topology as well as the anti-topology.

Importance of the Research Work

The study of topology on a set shows that a set can have multiple topologies of different types. The change in the conception of the notion of a set leads to an altogether different system of analysis. Such evolutions have already taken place with the conception of the fuzzy set which has led to a variety of study having many practical applications in many fields. Similar conclusions can be drawn with the generalization of the study of fuzzy set to the intuitionistic fuzzy set. Further, in the case of the neutrosophic set generalized the intuitionistic fuzzy set, the study in the field has revolutionized the study of the fuzzy set, leading to the expansion of analysis in real life situations, generalizing established algebras to algebras that should hold in reality. Such generalizations of the neutrosophic set led to the evolution of the neutro-topology and the anti-topology. It has been observed from literature that whenever any new topology is defined or introduced, the preliminary studies that are seen to be done in the new topology are on the interior, closure, exterior, boundary, continuity of functions, separation axioms and other basic aspects of topological spaces. Thus, since the neutro-topology and anti-topology have been introduced very lately and the preliminary studies had not been done by anyone, we have taken up the work in this research work. As such the preliminary study that has been undertaken in this study in the aspects of the interior, exterior, closure, boundary, continuity of functions and separation axioms with respect to the neutro-topology and anti-topology will be helpful in further studying the other aspects of the new topological spaces and expand the scope of analysis in the two areas.

Research Methodology

In this research work, the definition of neutro-topological space and anti-topological space are adopted and are used to study the properties. For the purpose of studying the properties in the new topological spaces, the properties of Interior, Exterior, Closure, and Boundary that are well established in general topological space will be used and corresponding studies will be made whether those properties hold in the context of the neutro-topology and the anti-topology. Further, properties of continuous functions that are already true in general topological space will be observed and corresponding studies will be made to evaluate their validity in the neutro-topology and the anti-topology. Further, separation axioms that are well established in general topological space will be considered to study the same with respect to the neutro-topology and the anti-topology. Finally, the concept of multisets will be borrowed and applied to the neutro-topology space and the anti-topology in analyzing some of the properties of Interior, Exterior, Closure and Boundary in multi-neutro-topological space, multi-neutro-Bi-topological space, and multi-anti-topological space.

Aims and Objectives

The survey of literature from point set theoretic topological spaces and its gradual development to fuzzy, intuitionistic fuzzy, neutrosophic and then to the neutro and anti-topologies shows that very little study has been made in the field of neutro-topological spaces and anti-topological spaces. Some work related to anti-topology has been done with regard to some of the preliminary properties of interior and closure. In the same work, the concept of anti-continuity has been defined in terms of the openness of the pre-image of open sets. We will use that definition to further analyze the properties of continuity, which has not been done or seen as done in literature. Many studies have been relatively done in the algebras of neutro and anti, as had been seen in literature, such as the definition and study of neutro-groups, neutro-rings, neutro-vector spaces, neutro-field and neutro-metric spaces. Also seen in literature are the studies on the anti-counterparts of the algebras like anti-groups and anti-rings. However, our study will not be along the line of the algebras but topological spaces. A study has been observed to have been done in group neutro-topological space and a study has also been done to find the number of neutro-topological spaces. Another study on ordered anti-topological spaces has been observed as done most recently. But studies on the interior, exterior, closure, and boundary has not been done in neutro-topological spaces. And even though preliminary studies of interior and closure has been done in anti-topological spaces, we will extend the study to add other properties in interior and closure in anti-topological spaces and further define exterior, and boundary in anti-topological spaces and study the corresponding properties. Continuity has been defined in the anti-topology without analyzing the properties of continuity in the space, we will study the properties of continuous functions in the anti-topological spaces by adopting the definition that had already been proposed. Further, we will study continuity in neutro-topological spaces by defining neutro-continuity in similar views of anti-continuity. We will also introduce a weaker form of continuity in neutro-topological spaces. Studies on the axioms of separation have not been done in either the neutro-topology or the anti-topology. We will define some axioms of separation in both the neutro-topological and anti-topological spaces and analyze the hereditary properties. Study of multisets have been done in fuzzy topology and in the neutrosophic topology, we will extend the classical set based neutro and anti-topologies to the theory of multisets and introduce multi-neutro and multi-anti topological spaces and study some properties of interior, closure, exterior and boundary. By the analysis of literature, for this research work, we have set the following objectives.

The objectives of the research work are:

(i) Observing and analysing the properties of Interior, Exterior, Closure, and Boundary points in neutro-topological spaces.

(ii) Defining neutro-bitopological spaces and observing and analysing the properties of Interior, Exterior, Closure, and Boundary points with respect to neutro-bitopological spaces.

(iii) Observing and analysing the properties of Interior, Exterior, Closure, and Boundary points in anti-topological spaces.

(iv) Study on continuous functions in neutro-topological, by introducing the concept of neutro-continuous functions and studying the properties of continuous functions in anti-topological spaces.

(v) Study on separation axioms in neutro-topological and anti-topological spaces.

(vi) Study on multi-neutro-topological spaces, multi-neutro-bitopological spaces and multi-anti-topological spaces.

Observing the proposed aims and objectives, the following chapters have been proposed.

CHAPTER-1: Introduction, Literature Review, and Basic Concepts.

CHAPTER-2: Properties of interior, exterior, closure, and boundary in neutro-topological spaces.

CHAPTER-3: Neutro-Bitopological Spaces.

CHAPTER-4: Study on Interior, Exterior, Closure, and Boundary in Anti-Topological spaces.

CHAPTER-5: Continuity of functions in Neutro-Topological and Anti-Topological Spaces

CHAPTER-6: Separation Axioms in Neutro-Topological and Anti-Topological Spaces

CHAPTER-7: Multi-Neutro-Topological Spaces, Multi-Neutro-Bi-Topological Spaces and Multi-Anti-Topological Spaces

CHAPTER-8: Summary and Conclusion

A brief discussion on what study has been undertaken in the chapters is given as follows:

CHAPTER-1: In the first introductory chapter, a detailed analysis has been provided from the origin of the study of topological spaces and how over the years the study in the subject has evolved. It has been discussed how fuzzy set and subsequently fuzzy topological spaces have been introduced. Further, it has been discussed how the fuzzy set has been extended to become the intuitionistic fuzzy set and hence the intuitionistic fuzzy topological space. Further, the development of the neutrosophic set and hence the neutrosophic topological space has been discussed, and further it has been discussed how the components of the neutrosophic set led to the development of the neutro-topology and the anti-topology. The aspects of interior, exterior, closure, boundary, continuity of functions, separation axioms, and the use of multiset theory have been discussed in all the different types of topological spaces. In this chapter, some basic concepts of topological spaces and some concepts in multisets have also been included.

CHAPTER-2: In this chapter, the aspects of interior, exterior, closure and boundary have been defined in neutro-topological spaces and have been named as neutro-interior, neutro-exterior, neutro-closure, and neutro-boundary. The various properties of all these aspects that are valid in a classical topology are analyzed with respect to the neutro-topology. Results that are valid in the neutro-topology have been provided with necessary proofs and necessary counter examples have been provided for those results which do not hold in neutro-topological spaces along with proper justifications.

CHAPTER-3: In this chapter, a neutro-bitopological space is introduced and the sets that are neutro-open in the space are considered to be neutro-open separately in the two neutro-topologies associated with the space. As such, no union or intersection of the neutro-open sets in the two neutro-topologies have been taken to define the neutro-openness in the neutro-bitopological space. Further, neutro-bi-interior, neutro-bi-exterior, neutro-bi-closure, and neutro-bi-boundary have been defined and their properties are analyzed. Results that are found to be valid have been provided with necessary proofs. Further, neutro-pseudo-exterior is defined after finding out that the neutro-bi-exterior, the way it is defined could not be analyzed for further results. In addition, neutro-quasi-interior and neutro-quasi-closure have also been defined and some of their properties are analyzed.

CHAPTER-4: In this chapter, the aspects of interior, exterior, closure and boundary have been defined in anti-topological spaces and have been named as anti-interior, anti-exterior, anti-closure, and anti-boundary. The various properties of all these aspects that are valid in a classical topology are analyzed in anti-topological spaces. Results that are valid in the anti-topology have been provided with necessary proofs and necessary counter examples have been provided for those results which do not hold in anti-topological spaces along with proper justifications.

CHAPTER-5: In this chapter, an attempt has been made to define continuity of functions in neutro-topological spaces. Properties of continuous functions in topological spaces have been analyzed to find which properties are valid in neutro-topological spaces. Further, taking advantage of the fact that a neutro-topology is obtainable from every classical topology, a new form of continuity, named as weakly neutro-continuity has been introduced and the properties of continuous functions have been compared in this form of continuity. Many results have been found to be valid in neutro-topological spaces with the new type of continuity. Necessary proofs have been provided for the propositions. Further, some analysis has been made on continuity in anti-topological spaces but only a few results could be established. The concept of weakly continuity could not be applied in this space.

CHAPTER-6: In this chapter, the axioms of separation T_0, T_1, T_2, regular space, T_3, normal space, and T_4 have been defined with respect to neutro-topology and anti-topology and have been named as neutro-T_0, neutro-T_1, neutro-T_2, neutro-regular, neutro-T_3, neutro-normal, neutro-T_4 and corresponding names have been given in the anti-topological space. Hereditary properties have been analyzed in both the spaces and the results that are true in the neutro and anti topological spaces have been provided with the necessary proofs. For the results which do not hold, justifications have been provided.

CHAPTER-7: In this chapter, the concept of multisets has been utilized to define multi-neutro-topological space, multi-neutro-bitopological space, and multi-anti-topological space. Further, multi-neutro-interior, multi-neutro-exterior, multi-neutro-closure, multi-neutro-boundary have been defined in multi-neutro-topological spaces and the properties of these aspects that have already been established in neutro-topological spaces have been extended to the multi-neutro-topological spaces. Similar study has also been done in multi-neutro-bitopological spaces and multi-anti-topological spaces.

CHAPTER-8: In this chapter, a summary of all the chapters have been provided along with the scope for future work in the field of neutro-topology and anti-topology.

Bibliography: Some references to the literature relevant to the abstract are given herewith.

Bibliography

[1] Agboola, A. A. A. (2020). Introduction to NeutroGroups. *International Journal of Neutrosophic Science*, **6**(1), 41-47.

[2] Agboola, A. A. A. (2020). Introduction to NeutroRings. *International Journal of Neutrosophic Science*, **7**(2), 62-73.

[3] Agboola, A. A. A. (2020). Introduction to AntiGroups. *International Journal of Neutrosophic Science*, **12**(2), 71-80.

[4] Agboola, A. A. A., & Ibrahim, M. A. (2020). Introduction to AntiRings. *Neutrosophic Sets and Systems*, **36**(1), 292-307.

[5] Atanassov, K. T. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, **20**(1), 87-96.

[6] Chang, C.L. (1968). Fuzzy topological spaces. *Journal of Mathematical Analysis and Applications*, **24**(1), 182-190.

[7] Coker, D. (1997). An Introduction to intuitionistic fuzzy topological spaces. *Fuzzy Sets and Systems*, **88**(1), 81-89.

[8] Hutton, B., & Reilly, I. (1980). Separation axioms in fuzzy topological spaces.8*Fuzzy Sets and Systems*, **3**, 93-104.

[9] Ibrahim, M. A., & Agboola, A. A. A. (2020). NeutroVector spaces I. *Neutrosophic Sets and Systems*, **36** (1), 328-350.

[10] Nazaroff, G. J. (1973). Fuzzy topological polysystems. *Journal of Mathematical Analysis and Applications*, **41**(2), 478-485.

[11] Sahin, M., Kargin, A., & Altun, A. (2021). Neutro-Metric spaces. *NeutroAlgebra Theory*, **1**, 71-84.

[12] Sahin, M., Kargin, A., & Yucel, M. (2021). Neutro-Topological space and Anti-Topological space. *NeutroAlgebra Theory*, **1**, 16-30.

[13] Salama, A. A., & Alblowi, S. A. (2012). Neutrosophic set and neutrosophic topological spaces. *IOSR Journal of Mathematics*, **3**(4), 31-35.

[14] Smarandache, F. (1998). Neutrosophy, neutrosophic probability, set, and logic. *ProQuest Information & Learning*. *Ann Arbor*, Michigan, USA, 105, 118-123.

[15] Smarandache, F. (2005). Neutrosophic set-a generalization of the intuitionistic fuzzy set. *International Journal of Pure and Applied Mathematics*, **24**(3), 287.

[16] Smarandache, F. (2019). Introduction to NeutroAlgebraic structures and AntiAlgebraic structures. *Advances of Standard and Nonstandard Neutrosophic Theories*, *Pons Edition*, Brussels, European Union.

[17] Smarandache, F. (2020). Introduction to NeutroAlgebraic structures and AntiAlgebraic structures (revisited). *Neutrosophic Sets and Systems*, **31**, 1-16.

[18] Smarandache, F. (2020). Generalizations and alternatives of classical algebraic structures to NeutroAlgebraic structures and AntiAlgebraic structures. *Journal of Fuzzy Extension & Applications*, **1**(2), 85.

[19] Smarandache, F. (2020). NeutroAlgebra is a generalization of partial algebra. *International Journal of Neutrosophic Science*, **2**(1), 8-17.

[20] Wong, C. K. (1974). Fuzzy points and local properties of fuzzy topology. *Journal of Mathematical Analysis and Applications*, **46**, 316-328.

[21] Yager, R. R. (1986). On the theory of bags. *International Journal of General Systems*, **13**(1), 23-37.

[22] Zadeh, L. A. (1975). The concept of a linguistic variable and its application to approximate reasoning-I. *Information Sciences*, **8**, 199-245.

List of Paper Publications:

1. Basumatary B., Khaklary, J.K., & Said B. (2022). On NeutroBitopological Spaces, *International Journal of Neutrosophic Science*, **18**(2), 254-261.
2. Basumatary B., Khaklary J.K., Wary N., & Smarandache F. (2022). On Neutro-Topological Spaces and their Properties. *Theory and Applications of NeutroAlgebras as Generalisations of Classical Algebras*. 180-201. *IGI Publications*.
3. Basumatary B., & Khaklary J.K. (2022). A Study on the Properties of Anti-Topological Space. 16-27. *Neutrosophic Algebraic Structures and Their Applications. NSIA Publications.*
4. Basumatary B., & Khaklary J.K. (2024). A Study on Continuity functions in neutro-topological spaces, *Neutrosophic Sets and Systems*, **78**, 341-352.